

Effects of Meson Mass Decrease on Superfluidity in Nuclear Matter

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Abstract

We calculate the 1S_0 pairing gap in nuclear matter by adopting the “in-medium Bonn potential” proposed by Rapp et al. [e-print nucl-th/9706006], which takes into account the in-medium meson mass decrease, as the particle-particle interaction in the gap equation. The resulting gap is significantly reduced in comparison with the one obtained by adopting the original Bonn potential.

Key words: Meson mass; Superfluidity; Nuclear matter

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Superfluidity in infinite hadronic matter has long been studied mainly in neutron matter from a viewpoint of neutron-star physics such as its cooling rates. As a way of description, relativistic models are attracting attention in addition to traditional non-relativistic nuclear many-body theories. Since Chin and Walecka succeeded in reproducing the saturation property of symmetric nuclear matter within the mean-field theory (MFT) [1], quantum hadrodynamics (QHD) which is an effective field theory of hadronic degrees of freedom has described not only infinite matter but also finite spherical, deformed and rotating nuclei successfully with various approximations [2,3]. These successes indicate that the particle-hole (p-h) channel in QHD is realistic. In contrast, relativistic nuclear structure calculations with pairing done so far have been using particle-particle (p-p) interactions borrowed from non-relativistic models such as Gogny force. Aside from practical successes of this kind of calculations, the p-p channel in QHD itself is a big subject which has just been started to study.

The first study of this direction was done by Kucharek and Ring [4]. They adopted, as the particle-particle interaction (v_{pp}) in the gap equation, a one-boson-exchange interaction with the ordinary relativistic MFT parameters, which gave the saturation under the no-sea approximation. The resulting maximum gap was about three times larger than the accepted values in the non-relativistic calculations. [5–9]. Various modifications to improve this result were proposed [10–12] but this has still been an open problem. One of such modifications is to include the p-h and the $N\bar{N}$ polarizations, the former of which is effective for reducing the gap in the non-relativistic models [13,14].

From a different viewpoint, Rummel and Ring [15,2] adopted the Bonn potential [16], which was constructed so as to reproduce the phase shifts of nucleon-nucleon scattering in free space, as v_{pp} and obtained pairing gaps consistent with the non-relativistic studies. Since the single particle states are determined by the MFT also in this calculation, explicit consistency between the p-h and

the p-p channels is abandoned. However, assuming that the MFT simulates the Dirac-Brueckner-Hartree-Fock (DBHF) calculation using the Bonn potential, we can consider that the consistency still holds implicitly. A possible extension of this study is to take into account the in-medium changes of the properties of the mesons which mediate the inter-nucleon forces. Among them, the change of the mass, that is, the momentum-independent part of the self-energy, has been discussed in terms of the Brown–Rho (BR) scaling [17], the QCD sum rules [18], and some other models both in the quark level and in the hadron level [19,20]. Although there still has been theoretical controversy [21], some experiments look to support the vector meson mass decrease [22]. In addition, the momentum-dependent part also attracts attention recently [23–26]. In the present work, we assume only the momentum-independent vector meson mass decrease. A simple way to take into account this meson mass decrease in the meson exchange interactions is to assume the BR scaling. Actually Rapp et al. [27] showed, based on this, that the saturation property of symmetric nuclear matter and the mass decrease of the vector mesons were compatible. So we adopt their “in-medium Bonn potential” as v_{pp} in the gap equation. Since the gap equation takes the form such that the short range correlation is involved [28,29,7], we use this interaction in the gap equation without the iteration to construct the G -matrix.

As described in ref. [4], meson fields also have to be treated dynamically beyond the MFT to incorporate the pairing field via the anomalous (Gor’kov) Green’s functions [30]. The resulting Dirac-Hartree-Fock-Bogoliubov equation reduces to the ordinary BCS equation in the infinite matter case. We start from a model Lagrangian for the nucleon, the σ boson, and the ω meson,

$$\begin{aligned}
\mathcal{L} = & \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi \\
& + \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\
& + g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi, \\
F_{\mu\nu} = & \partial_\mu\omega_\nu - \partial_\nu\omega_\mu.
\end{aligned} \tag{1}$$

The actual task is to solve the coupled equations for the effective nucleon mass and the 1S_0 pairing gap:

$$\begin{aligned}
M^* &= M - \frac{g_\sigma^2}{m_\sigma^2} \frac{\gamma}{2\pi^2} \int_0^{\Lambda_c} \frac{M^*}{\sqrt{\mathbf{k}^2 + M^{*2}}} v^2(k) k^2 dk, \\
\Delta(p) &= -\frac{1}{8\pi^2} \int_0^{\Lambda_c} \bar{v}_{pp}(p, k) \frac{\Delta(k)}{\sqrt{(e_k - e_{k_F})^2 + \Delta^2(k)}} k^2 dk, \\
v^2(k) &= \frac{1}{2} \left(1 - \frac{e_k - e_{k_F}}{\sqrt{(e_k - e_{k_F})^2 + \Delta^2(k)}} \right), \\
e_k &= \sqrt{\mathbf{k}^2 + M^{*2}} + g_\omega \langle \omega^0 \rangle,
\end{aligned} \tag{2}$$

where $\bar{v}_{pp}(p, k)$ is an anti-symmetrized matrix element of the adopted p-p interaction,

$$\begin{aligned}
&\bar{v}_{pp}(\mathbf{p}, \mathbf{k}) \\
&= \langle \mathbf{p}s', \tilde{\mathbf{p}}s' | v_{pp} | \mathbf{k}s, \tilde{\mathbf{k}}s \rangle - \langle \mathbf{p}s', \tilde{\mathbf{p}}s' | v_{pp} | \tilde{\mathbf{k}}s, \mathbf{k}s \rangle,
\end{aligned} \tag{3}$$

with an instantaneous approximation (energy transfer = 0) and an integration with respect to the angle between \mathbf{p} and \mathbf{k} to project out the S -wave component, and $\langle \omega^0 \rangle$ is determined by the baryon density. Here we consider only symmetric nuclear matter ($\gamma = 4$) and neutron matter ($\gamma = 2$). Conceptually the numerical integrations run to infinity in the present model where the finiteness of the hadron size is considered in v_{pp} . Numerically, however, we introduce a cut-off Λ_c ; its value is chosen to be 20 fm^{-1} so that the integrations converge. If we adopt the Bonn potential as v_{pp} and include the ρ meson and the cubic and quartic self-interactions of σ with choosing the NL1 parameter set [31] for the mean field, these equations reproduce the results of ref. [15]. Here we note that there are pros and cons as for the self-interaction terms [32,33]. First we developed a computer code to calculate the Bonn potential for this channel from scratch, and later confirmed that our code reproduced outputs of the one in ref. [34].

Then in the present work we adopt the in-medium Bonn potential of Rapp et al. [27] although this is, as yet, unpublished. To construct this potential, they applied the BR scaling after replacing the σ boson in the original Bonn-B potential with the correlated and the uncorrelated 2π exchange processes. Then they parameterized the obtained nucleon-nucleon interaction by three scalar bosons σ_1 , σ_2 and “rest- σ ”, in addition to π , η , ρ , ω and δ . Here σ_1 and σ_2 with density-dependent masses and coupling constants simulate the correlated 2π processes and the “rest- σ ” simulates the uncorrelated ones. Therefore the actual tasks are adding two extra scalar bosons to the original Bonn-B potential and applying the BR scaling,

$$\frac{M^*}{M} = \frac{m_{\rho,\omega}^*}{m_{\rho,\omega}} = \frac{\Lambda_{\rho,\omega}^*}{\Lambda_{\rho,\omega}} = 1 - C \frac{\rho}{\rho_0}, \quad (4)$$

where ρ_0 is the normal nuclear matter density, $\Lambda_{\rho,\omega}$ are the cut-off masses in the form factors of the meson-nucleon vertices, which is related to the hadron size. The scaling parameter C is chosen to be 0.15. Aside from a slight tuning of the coupling constant of δ , other parameters are the same as those in the original Bonn-B potential. All the potential parameters are presented in Tables I and II in ref.[27]. Note that this scaling applies only to the quantities in v_{pp} , since the MFT with its effective nucleon mass guarantees the saturation and we confirmed that the effects of pairing on the bulk properties are negligible except at very low densities in the present model. As for the π -N coupling in the potential, we adopt the pseudovector one conforming to the suggestion of chiral symmetry [35] and actually the pseudoscalar one in the Bonn potential was shown to lead to unrealistically attractive contributions in the DBHF calculation [36].

The result is presented in Fig.1. This shows that applying the BR scaling reduces the gap significantly in comparison with the original Bonn-B potential. The potentials themselves are given in Fig.2. The relation between the reduction of the gap and the change in the potential, i.e., the shift to lower momenta,

can be understood as follows: As discussed in refs. [6,15], $\Delta(k)$ determined by the second equation of (2) exhibits a nodal structure similar to $v_{pp}(k_F, k)$ as functions of k with the opposite sign and some possible deviation of zeros. Consequently both the low-momentum attractive part where $\Delta(k) > 0$ and the high-momentum repulsive part where $\Delta(k) < 0$ give positive contributions to $\Delta(k_F)$, and Fig.2 shows that the both of these contributions are reduced in the case of the in-medium Bonn potential. This is the main reason why $\Delta(k_F)$ is reduced by applying the BR scaling. Actually we confirmed that this is mainly due to the mass decrease of the vector mesons, not of the nucleon, as shown in Fig.3 although also the latter produces some reduction of the gap. Note here that the saturation in the DBHF calculation is guaranteed only when both M and $m_{\rho,\omega}$ are scaled. The in-medium meson mass decrease is brought about by the N- \bar{N} polarization in hadronic models. Although its explicit inclusion in the gap equation has not been reported yet, it is conjectured in ref. [12] that its inclusion simultaneously with the p-h one will reduce the gap as that of the p-h one in the non-relativistic models [13,14]. In this sense, the present result that applying the BR scaling reduces the gap is consistent with this conjecture, although the reduction in refs.[13,14] is more drastic than that in the present work. An additional element is that the cut-off masses in the form factors of the meson-nucleon vertices are also scaled. Since they appear in the form

$$\frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + \mathbf{q}^2}, \quad (5)$$

where \mathbf{q} is the momentum transfer [16], simultaneous reductions of Λ_α and m_α ($\alpha = \rho, \omega$), lead to a further reduction of the repulsion due to the vector mesons at large- $|\mathbf{q}|$. Since the higher the background density becomes the more nucleons feel the high-momentum part of v_{pp} , the in-medium reduction of $\Lambda_{\rho,\omega}$ leads to an additional reduction of the gap in the high-density region.

Finally, use of the NL1 parameter set for the mean field brings negligible

changes and the result for neutron matter is very similar to those presented here for symmetric nuclear matter.

To summarize, we adopted the in-medium Bonn potential, proposed by Rapp et al., which takes into account the in-medium meson mass decrease in a simple form and is compatible with the nuclear matter saturation in the DBHF calculation, as the particle-particle interaction in the gap equation. The resulting pairing gap in nuclear matter is significantly reduced in comparison with the one obtained by adopting the original Bonn potential. Here we should note that the uncertainty in the gap value deduced from various studies is still larger than the strength of the reduction found in the present work, and therefore, further investigations are surely necessary both relativistically and non-relativistically.

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Figure Captions

Fig.1. Pairing gap in symmetric nuclear matter at the Fermi surface as functions of the Fermi momentum. Dotted and solid lines indicate the results obtained by adopting the Bonn-B and the in-medium Bonn potentials, respectively. Note that accuracy is somewhat less around $\Delta(k_F) \simeq 0$ in our code.

Fig.2. Matrix element $\bar{v}_{pp}(k_F, k)$ as functions of the momentum k , with a Fermi momentum $k_F = 0.9 \text{ fm}^{-1}$. Dotted and solid lines indicate the results obtained by adopting the Bonn-B and the in-medium Bonn potentials, respectively.

Fig.3. Pairing gap in symmetric nuclear matter at the Fermi surface as functions of the Fermi momentum. Solid, dotted and dashed lines indicate that the results obtained by reducing according to Eq. (4) only the nucleon mass, only the vector meson masses, and the both, respectively.