

Chiral phase transition in an extended NJL model with higher-order multi-quark interactions

Kouji Kashiwa,¹ Hiroaki Kouno,^{2,*} Tomohiko Sakaguchi,¹

Masayuki Matsuzaki,³ and Masanobu Yahiro¹

¹*Department of Physics, Graduate School of Sciences,
Kyushu University, Fukuoka 812-8581, Japan*

²*Department of Physics, Saga University, Saga 840-8502, Japan*

³*Department of Physics, Fukuoka University of Education,
Munakata, Fukuoka 811-4192, Japan*

Abstract

The chiral phase transition is studied in an extended Nambu–Jona-Lasinio model with eight-quark interactions. Equations for scalar and vector quark densities, derived in the mean field approximation, are nonlinear and mutually coupled. The scalar type nonlinear term hastens the restoration of chiral symmetry, while the scalar-vector mixing term makes the transition sharper. The scalar type nonlinear term shifts the critical endpoint toward the values predicted by lattice QCD simulations and the QCD-like theory.

PACS numbers: 11.30.Rd, 12.40.-y

*kounoh@cc.saga-u.ac.jp

Qualitative properties in quantum chromodynamics (QCD) at high temperatures and densities attract much attention. One of the most important recent findings is strong correlations in the quark gluon plasma just above the critical temperature; it is realized as the near perfect fluidity [1–3] and the mesonic correlations [4, 5].

With the aid of the progress in computer power, lattice QCD simulations have become feasible for thermal systems at zero or small density [6, 7]. At high density, however, lattice QCD is still not feasible due to the sign problem. As an approach complementary to the first-principle lattice simulations, we can consider several effective models. One of them is the Nambu–Jona-Lasinio (NJL) model. Since it was proposed [8], although this is a model of chiral symmetry that does not possess a confinement mechanism, this model has been widely used [9, 10] in the mean field approximation (MFA), for example, for analyses of the critical endpoint of chiral transition [11–16].

Although the NJL model is recognized as an useful method for understanding the chiral symmetry breaking, only a few studies were done so far on roles of higher-order multi-quark interactions [17, 18], except for the case of the six-quark interaction coming from the 't Hooft determinant interaction [19]. Since the NJL model is an effective theory of QCD, there is no reason, in principle, why higher-order multi-quark interactions are excluded. In the nonperturbative renormalization group method [20], such higher-order interactions are produced as a result of quantum effects in the high momentum region. Such effects should be included in the low-energy effective action from the beginning. Thus, it is quite meaningful to study effects of higher-order interactions on the chiral phase transition.

In this paper, we consider an extended NJL model that newly includes eight-quark interactions and analyze roles of such higher-order interactions on the chiral phase transition. It is well known that the original NJL model predicts a critical endpoint to appear at a lower temperature (T) and a higher chemical potential (μ) than lattice QCD [7] and the QCD-like theory [21, 22] do. We will show in this paper that a scalar type eight-quark interaction shifts the critical endpoint toward values predicted by lattice simulations and the QCD-like theory.

As for the repulsive vector type four-quark interaction $(\bar{q}\gamma_\mu q)^2$, it is well-known that it makes the chiral phase transition weaker in the low T and high μ region, so there is a possibility that the transition becomes a crossover in the region when the interaction is strong enough [12, 16]. In this point of view, an *absence* of the vector type four-quark

interaction may be preferable in the high density region; the absence is also supported by works of Refs. [23] and [24].

On the contrary, in the relativistic meson-nucleon theory [25], the repulsive force mediated by vector mesons is necessary to account for the saturation property of nuclear matter. Using the auxiliary field method, we can convert quark-quark interactions to meson-quark interactions; for example, see Refs. [26–28] and references therein. It is then natural to think that there exists a relation between the meson-nucleon interaction and the quark-quark interaction in the NJL model, since a nucleon is composed of three constituent quarks each of which has a large effective mass as a result of the spontaneous symmetry breaking (SSB) of chiral symmetry. In this point of view, the vector type four-quark interaction $(\bar{q}\gamma_\mu q)^2$ is *indispensable* around $\mu \sim 308$ MeV corresponding to the normal nuclear density region. Thus, it is expected that the vector type interaction is sizable in the normal density region but suppressed in the higher density region.

In the relativistic meson-nucleon theory, it is known that nonlinear meson-nucleon interactions suppress the effective coupling between mesons and nucleons in the higher density region [28–31]. It is then strongly expected that a similar situation takes place in the NJL model as soon as higher-order multi-quark interactions are introduced. This is shown in this paper.

This is the first work to study roles of eight-quark interactions on phase transitions, so we concentrate our analysis on the chiral phase transition and use a simple model with two flavor quarks. Effects of the higher-order interactions on color superconductivity will be discussed in a forthcoming paper.

We start with the following chiral-invariant Lagrangian density with two flavor quarks

$$\begin{aligned} \mathcal{L} = & \bar{q}(i\not{\partial} - m_0)q + \left[g_{2,0} \left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 \right) + g_{4,0} \left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 \right)^2 \right. \\ & \left. - g_{0,2} (\bar{q}\gamma^\mu q)^2 - g_{2,2} \left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 \right) (\bar{q}\gamma^\mu q)^2 + \cdots \right], \end{aligned} \quad (1)$$

where q , m_0 and $g_{i,j}$ stand for a quark field, a bare quark mass and coupling constants. Here, we consider only four- and eight-quark interactions by ignoring the higher-order interactions denoted by dots in Eq. (1). Furthermore, we disregard interactions including isovector-vector current that are not important in symmetric quark matter, and also the vector type eight-quark interaction $(\bar{q}\gamma_\mu q)^4$, because the expectation value of the vector current $\bar{q}\gamma_0 q$ is smaller than that of the scalar one $\bar{q}q$, unless the chemical potential is quite large. The mean

field approximation reduces the Lagrangian to

$$\begin{aligned}
\mathcal{L}_{\text{MFA}} = & \bar{q}(i \not{\partial} - m_0)q \\
& + \left[2g_{2,0} + 4g_{4,0} \left(\langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) - 2g_{2,2} \langle \bar{q}\gamma^\mu q \rangle^2 \right] \left(\langle \bar{q}q \rangle \bar{q}q + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle \bar{q}i\gamma_5 \vec{\tau}q \right) \\
& - \left[2g_{0,2} + 2g_{2,2} \left(\langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) \right] \langle \bar{q}\gamma^\mu q \rangle \bar{q}\gamma_\mu q \\
& - g_{2,0} \left(\langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) - 3g_{4,0} \left(\langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right)^2 \\
& + 3g_{2,2} \left(\langle \bar{q}q \rangle^2 + \langle \bar{q}i\gamma_5 \vec{\tau}q \rangle^2 \right) \langle \bar{q}\gamma^\mu q \rangle^2 + g_{0,2} \langle \bar{q}\gamma^\mu q \rangle^2.
\end{aligned} \tag{2}$$

Below, just for simplicity of the notation, we will omit terms including the pseudo-scalar current $\bar{q}i\gamma_5 \vec{\tau}q$ and the spatial components $\bar{q}\gamma_i q$ ($i = 1, 2, 3$) of the vector current, since their expectation values vanish. It is convenient to introduce two auxiliary mean fields as

$$\sigma \equiv \langle \bar{q}q \rangle, \quad \omega \equiv \langle \bar{q}\gamma_0 q \rangle. \tag{3}$$

Using these auxiliary fields, one can rewrite \mathcal{L}_{MFA} as

$$\mathcal{L}_{\text{MFA}} = \bar{q}[i \not{\partial} - (m_0 + \Sigma_s) + \Sigma_v \gamma_0]q - U, \tag{4}$$

where

$$\begin{aligned}
\Sigma_s &= -\left(2g_{2,0}\sigma + 4g_{4,0}\sigma^3 - 2g_{2,2}\sigma\omega^2 \right), \quad \Sigma_v = -\left(2g_{0,2}\omega + 2g_{2,2}\sigma^2\omega \right), \\
U &= g_{2,0}\sigma^2 + 3g_{4,0}\sigma^4 - 3g_{2,2}\sigma^2\omega^2 - g_{0,2}\omega^2.
\end{aligned} \tag{5}$$

It should be noted that the fourth-order scalar interaction $(\bar{q}q)^4$ with positive sign in Eq. (1) turns out to be the σ^4 term with positive sign in the mesonic potential U . This model is thus stable against big fluctuations in σ , as long as $g_{4,0} \geq 0$.

The thermodynamical potential Ω of the system with finite T and μ is then given by

$$\begin{aligned}
\Omega(T, \mu) = & -2N_f N_c V \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[E_p + \frac{1}{\beta} \left(\log(1 + e^{-\beta(E_p - \mu^*)}) + \log(1 + e^{-\beta(E_p + \mu^*)}) \right) \right] \\
& + VU,
\end{aligned} \tag{6}$$

where $\beta = 1/T$, $\mu^* = \mu + \Sigma_v$, $E_p = \sqrt{\mathbf{p}^2 + M^2}$ and M stands for the effective quark mass defined as $M = m_0 + \Sigma_s$. The corresponding scalar and vector quark densities, ρ_s and ρ_v , are also given by

$$\rho_s = 2N_f N_c \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{M}{E_p} (n_q + n_{\bar{q}} - 1), \quad \rho_v = 2N_f N_c \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (n_q - n_{\bar{q}}), \tag{7}$$

where $n_q = 1/[1 + \exp\{\beta(E_p - \mu^*)\}]$ and $n_{\bar{q}} = 1/[1 + \exp\{\beta(E_p + \mu^*)\}]$. The physical solutions of σ and ω satisfy the stationary condition

$$\begin{pmatrix} \frac{\partial}{\partial \sigma} \left(\frac{\Omega}{V} \right) \\ \frac{\partial}{\partial \omega} \left(\frac{\Omega}{V} \right) \end{pmatrix} = \mathcal{G}^* \begin{pmatrix} \sigma - \rho_s \\ \rho_v - \omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathcal{G}^* \equiv \begin{pmatrix} G_{s\sigma}^* & G_{v\sigma}^* \\ G_{s\omega}^* & G_{v\omega}^* \end{pmatrix}. \quad (8)$$

Here we have defined four effective couplings, $G_{s\sigma}^*$, $G_{v\sigma}^*$, $G_{s\omega}^*$ and $G_{v\omega}^*$, as [28–31]

$$G_{s\sigma}^* \equiv -\frac{\partial \Sigma_s}{\partial \sigma} = 2g_{2,0} + 12g_{4,0}\sigma^2 - 2g_{2,2}\omega^2, \quad G_{v\sigma}^* \equiv -\frac{\partial \Sigma_v}{\partial \sigma} = 4g_{2,2}\sigma\omega. \quad (9)$$

$$G_{s\omega}^* \equiv -\frac{\partial \Sigma_s}{\partial \omega} = -4g_{2,2}\sigma\omega = -G_{v\sigma}^*, \quad G_{v\omega}^* \equiv -\frac{\partial \Sigma_v}{\partial \omega} = 2g_{0,2} + 2g_{2,2}\sigma^2. \quad (10)$$

When $\det(\mathcal{G}^*) = G_{s\sigma}^* G_{v\omega}^* + G_{v\sigma}^{*2} \neq 0$, which is satisfied in our analyses below, the matrix \mathcal{G}^* has its inverse, and then the stationary condition (8) leads to $\sigma = \rho_s$ and $\omega = \rho_v$, showing the consistency with Eq.(3) [30]. However, the solutions σ and ω to the equations do not always yield a minimum of Ω . The solution σ is then determined by searching for minima of $\Omega(\sigma, \omega(\sigma))$ in which ω is eliminated using $\omega = \rho_v$.

Identifying $\bar{q}i\gamma_5\vec{\tau}q$ with the pion field, we can define the effective coupling $G_{s\pi}^*$ between a pion and quarks as

$$iG_{s\pi}^* \gamma_5 \vec{\tau} \equiv \frac{\delta^3}{\delta q \delta \langle \bar{q}i\gamma_5\vec{\tau}q \rangle \delta \bar{q}} \mathcal{L}_{\text{MFA}} \Big|_{\langle \bar{q}i\gamma_5\vec{\tau}q \rangle=0} = i(2g_{2,0} + 4g_{4,0}\sigma^2 - 2g_{2,2}\omega^2) \gamma_5 \vec{\tau}. \quad (11)$$

In the random phase approximation, the pion mass M_π at $T = \mu = 0$ is determined with this effective coupling as

$$M_\pi^2 = \frac{4m_0}{G_{s\pi}^* MI(M, M_\pi)}, \quad I(x, y) = 8N_f N_c \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{\mathbf{p}^2 + x^2} (4(\mathbf{p}^2 + x^2) - y^2)}. \quad (12)$$

Similarly, the σ -meson mass M_σ at $T = \mu = 0$ is determined as

$$M_\sigma^2 = \frac{4m_0 + 32g_{4,0}\sigma^3}{G_{s\sigma}^* MI(M, M_\sigma)} + 4M^2. \quad (13)$$

This equation indicates that $M_\sigma < 2M$ when $\sigma < 0$ and $g_{4,0} > 0$.

Since the NJL model is nonrenormalizable, it is needed to introduce a cutoff in the momentum integration. In this paper, we use the three-dimensional momentum cutoff as

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \rightarrow \frac{1}{2\pi^2} \int_0^\Lambda p^2 dp. \quad (14)$$

Hence, the present model has six parameters, m_0 , Λ , $g_{2,0}$, $g_{4,0}$, $g_{0,2}$ and $g_{2,2}$. We simply assume $m_0 = 5.5$ MeV. For the case of $g_{4,0} = 0$, Λ and $g_{2,0}$ are fixed to reproduce the empirical

values of the pion decay constant $f_\pi = 93.3$ MeV and the pion mass $M_\pi = 138$ MeV. For the case of nonzero $g_{4,0}$, parameters Λ , $g_{2,0}$ and $g_{4,0}$ are fixed to reproduce f_π and M_π mentioned above and $M_\sigma = 650$ MeV. In the latter case, the contribution of the $g_{4,0}$ term to $G_{s\sigma}^*$ is about 11 percents at $T = 0$ and $\mu = 0$. For the vector coupling constants $g_{0,2}$ and $g_{2,2}$, we take two extreme cases, $G_\omega = 0$ and $G_\omega = G_\sigma/1.5$, where $G_\sigma \equiv G_{s\sigma}^*|_{T=\mu=0} = 2g_{2,0} + 12g_{4,0}\sigma_0^2$ and $G_\omega \equiv G_{v\omega}^*|_{T=\mu=0} = 2g_{0,2} + 2g_{2,2}\sigma_0^2$ for σ_0 the scalar density at $T = 0$ and $\mu = 0$. Furthermore, in order to determine each value of $g_{0,2}$ and $g_{2,2}$, we take two cases, $(2g_{0,2}, 2g_{2,2}\sigma_0^2) = (G_\omega, 0)$ and $(2g_{0,2}, 2g_{2,2}\sigma_0^2) = (0.8G_\omega, 0.2G_\omega)$. For the second case, $G_{v\omega}^*$ is suppressed in the high μ region, as shown later. Table I summarizes the parameter sets we have taken, M_σ at $T = 0$ and $\mu = 0$, and the type of transition at $T = 0$.

model	$g_{2,0}$	$g_{4,0}\sigma_0^2$	$2g_{0,2}$	$2g_{2,2}\sigma_0^2$	M_σ [GeV]	type
NJL	5.498	0	0	0	0.681	first order
NJL + σ^4	5.276	0.1109	0	0	0.650	first order
NJL + ω^2	5.498	0	G_ω	0	0.681	crossover
NJL + $\omega^2 + \sigma^2\omega^2$	5.498	0	$0.8 G_\omega$	$0.2 G_\omega$	0.681	crossover
NJL + $\sigma^4 + \omega^2$	5.276	0.1109	G_ω	0	0.650	crossover
NJL + $\sigma^4 + \omega^2 + \sigma^2\omega^2$	5.276	0.1109	$0.8 G_\omega$	$0.2 G_\omega$	0.650	first order

TABLE I: Summary of the parameter sets, M_σ at $T = 0$ and $\mu = 0$, and the type of the transition at $T = 0$. The coupling constants are shown in GeV^{-2} . The effective coupling G_ω is fixed to 7.331 GeV^{-2} in the NJL+ ω^2 and the NJL+ $\omega^2+\sigma^2\omega^2$ model and to 7.922 GeV^{-2} in the NJL+ $\sigma^4+\omega^2$ and the NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$ model. For all cases, we take $\Lambda = 0.6315$ GeV and $M|_{T=\mu=0} = 0.3379$ GeV and $\sigma_0 = -0.03023 \text{ GeV}^3$.

Figure 1(a) shows the T dependence of the effective quark mass for the case of $\mu = 0$. Since the ω field has no contribution to M when $\mu = 0$, results are shown only for the original NJL and the NJL+ σ^4 model. The nonlinear σ^4 interaction makes the restoration of chiral symmetry faster, since the effective coupling, responsible for the SSB of chiral symmetry, becomes smaller as T increases, as shown in Fig.1(b). However, the interaction keeps the transition a crossover as in the case of the original NJL. The NJL+ σ^4 model yields a smaller transition temperature ($T_c \sim 180 \text{ MeV}$) than the original NJL model does ($T_c \sim 190 \text{ MeV}$). The value, $T_c \sim 180 \text{ MeV}$, is close to the one predicted by the lattice simulation ($T_c \sim 170 \text{ MeV}$) [7].

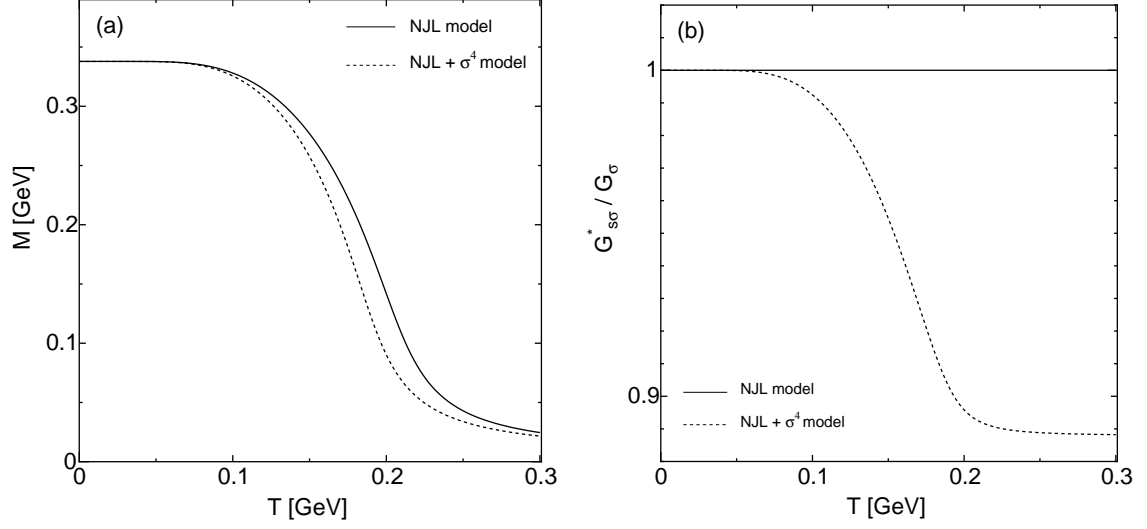


Fig. 1: The T dependence of (a) the effective quark mass and (b) the effective coupling in the case of $\mu = 0$.

Figure 2 shows the effective quark mass as a function of μ with $T = 0$. In the original NJL model, the chiral transition is a first order. The nonlinear σ^4 interaction makes the transition faster, as shown in the result of the NJL+ σ^4 model.

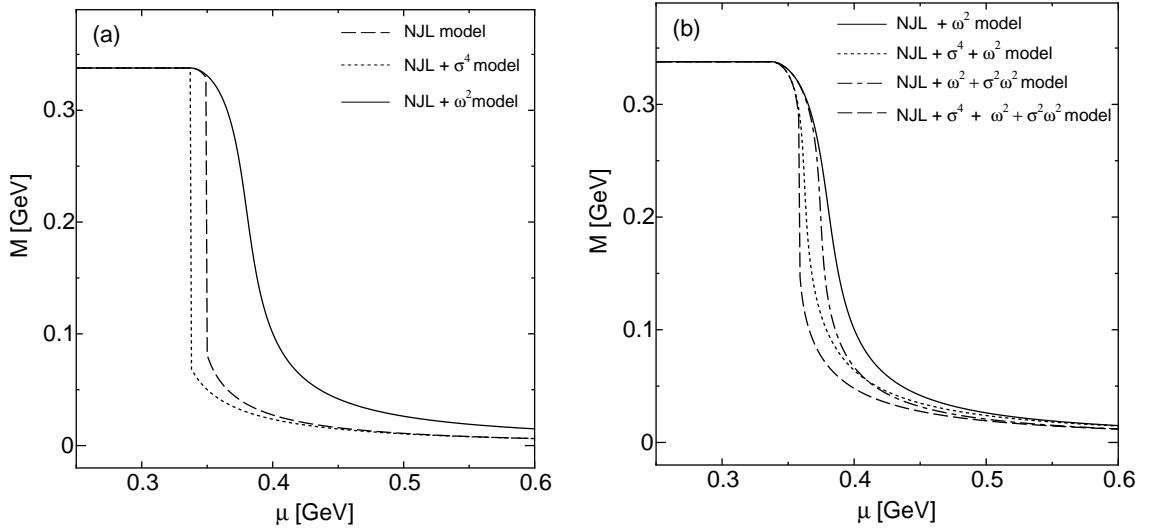


Fig. 2: The μ dependence of the effective quark mass in the case of $T = 0$.

As already shown in Refs. [12, 16], the ω^2 interaction tends to change the transition from

a first order to a crossover; actually this is seen in the result of the NJL+ ω^2 model of Fig. 2(a). As an interesting result, however, the nonlinear σ^4 and $\sigma^2\omega^2$ interactions make the transition sharper again, so that the transition returns to a first order in the full-fledged NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$ model with the present parameter set. Thus, the nonlinear σ^4 and $\sigma^2\omega^2$ interactions work so as to cancel out the well-known effect of the ω^2 interaction.

Figure 3 shows the effective couplings as functions of μ for the case of $T = 0$. For all models, both $G_{s\sigma}^*$ and $G_{v\omega}^*$ are suppressed in the high μ region, but each model has its own μ dependence. In the NJL+ σ^4 model, $G_{s\sigma}^*$ decreases suddenly in the high μ region and then approaches the value of $2g_{2,0}$, because the σ -dependent part of $G_{s\sigma}^*$ almost vanishes there. Similarly, in the NJL+ $\sigma^4+\omega^2$ model, $G_{s\sigma}^*$ decreases rather rapidly but not suddenly as μ increases and approaches the value of $2g_{2,0}$. The change in the μ dependence of $G_{s\sigma}^*$ from the NJL+ σ^4 model to the NJL+ $\sigma^4+\omega^2$ model comes from the fact that the ω^2 interaction makes the phase transition weak. In the NJL+ $\omega^2+\sigma^2\omega^2$ model, $G_{s\sigma}^*$ decreases gradually as μ increases, because the ω -dependent part of $G_{s\sigma}^*$ gives a negative contribution to the effective coupling. In the full-fledged NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$ model, $G_{s\sigma}^*$ is suppressed suddenly by the σ^4 interaction and then suppressed gradually by the $\sigma^2\omega^2$ mixing interaction.

As for $G_{v\omega}^*$ shown in Fig. 3(b), one can see a similar sharp suppression but not find any gradual decrease. This is understandable from the fact that in Eq. (10) the coupling has a σ -dependent term but no ω -dependent one. As a point to be noted, the sharp suppression comes from the $\sigma^2\omega^2$ interaction for the case of $G_{v\omega}^*$ but from the σ^4 interaction for the case of $G_{s\sigma}^*$. Furthermore, note that the $\sigma^2\omega^2$ interaction yields both a gradual decrease of $G_{s\sigma}^*$ and a sharp suppression of $G_{v\omega}^*$.

The μ dependence of the effective couplings mentioned above is similar to that of the chiral symmetry restoration shown in Fig. 2. We can then consider that effects of higher-order interactions on the chiral symmetry restoration are described mainly through the effective couplings in their μ dependences, although U is also changed by the higher-order interactions.

Figure 4 shows the phase diagram in the μ - T plane. Results are shown for the three cases of the original NJL model, the NJL+ σ^4 model and the NJL+ $\sigma^4+\omega^2+\sigma^2\omega^2$ model in which the transitions are first order in the high μ and low T region. First-order transitions take place on the curves in Fig. 4, and at the endpoint (μ_e, T_e) of each curve the transition is changed into a crossover. The NJL+ σ^4 model yields smaller μ_e and larger T_e than the original

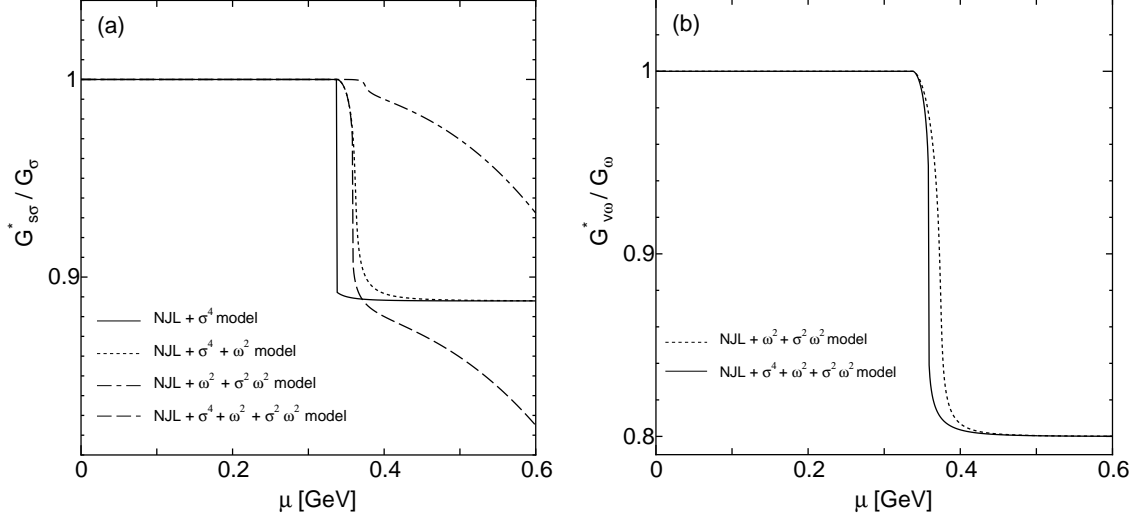


Fig. 3: Effective couplings as functions of μ in the case of $T = 0$.

NJL model does; $(\mu_e, T_e) = (308, 54)$ MeV for the former and $(330, 47)$ MeV for the latter. When we take another parameter set, $\Lambda = 0.6315$ GeV, $g_{2,0} = 5.00$ GeV $^{-2}$ and $g_{4,0} = 271$ GeV $^{-8}$, that reproduces $M_\sigma = 600$ MeV, the endpoint of the NJL+ σ^4 model is at $(276, 62)$ MeV; see the dot-dashed curve in Fig. 4. Thus, the nonlinear σ interaction shifts the critical endpoint toward values predicted by a lattice QCD calculation, $(\mu_e, T_e) = (242, 160)$ MeV [7], and by the QCD-like theory, $(\mu_e, T_e) \sim (200, 100)$ MeV [21, 22].

As mentioned above, the ω^2 interaction tends to change the transition from a first order to a crossover, but this effect is partially canceled out by the scalar-vector mixing interaction $\sigma^2 \omega^2$. Consequently, as shown in Fig. 4, there exists a critical endpoint also in the NJL+ $\sigma^4 + \omega^2 + \sigma^2 \omega^2$ model. A comparison of our models with the results of lattice simulations and the QCD-like theory indicates that the NJL+ σ^4 model is most preferable among our models. This may imply that the cancellation is essential.

As pointed out in Ref. [12], in the ordinary NJL model with no higher-order interaction, the quark matter equation of state has a saturation point, as far as the vector interaction is not too strong. However, the resulting baryon number density ρ_{sat} at the saturation point is rather higher than the normal density ρ_0 of nuclear matter, say $\rho_{\text{sat}} \sim 2\rho_0$. When the vector interaction is quite strong, there exists no saturation point. In our model, due to the higher-order interactions such as σ^4 and/or $\sigma^2 \omega^2$, there can be a saturation point, even if the vector interaction is rather strong. However, the saturation density is still higher than

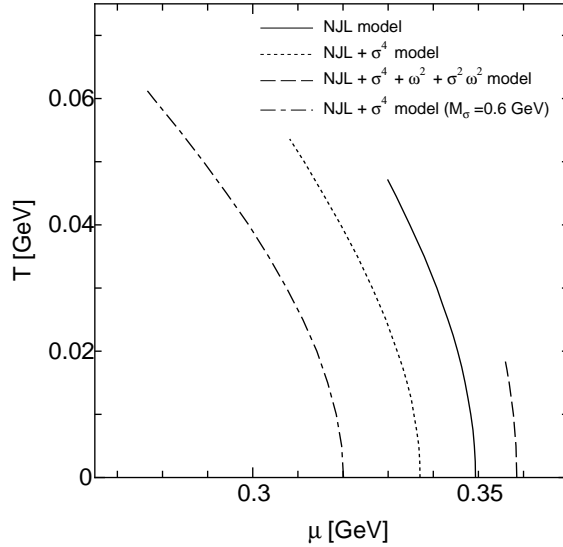


Fig. 4: Phase diagram in the μ - T plane. Each curve represents the location of the first-order phase transition in each model. First three curves are the results of the models with the parameter set that gives $M_\sigma = 0.65$ GeV, while the last one is the result of the model with the parameter set that gives $M_\sigma = 0.6$ GeV at the vacuum.

ρ_0 . This deviation may come from the fact that the present model has no confinement.

In the present model, the temperature of the critical endpoint is still lower than that obtained by lattice QCD, even if the higher-order interactions are taken into account. The deviation may be originated from confinement and the hadronic properties not included in the present model. It is reported that baryon resonances make the temperature of the critical endpoint much higher in the chiral hadron model [32, 33].

In summary, we have studied effects of eight-quark interactions on the chiral phase transition. The scalar type nonlinear interaction σ^4 hastens the restoration of chiral symmetry and shifts the critical endpoint toward the values predicted by lattice simulations and the QCD-like theory. The scalar-vector mixing interaction $\sigma^2 \omega^2$ can make the transition sharper in the high μ and low T region, while the ω^2 interaction works in the opposite direction. Thus, the effect of the mixing interaction tends to cancel out that of the ω^2 interaction in the high μ region, while the ω^2 interaction is still strong in the normal nuclear density region. The roles of the eight-quark interactions are well understood through the effective couplings, $G_{s\sigma}^*$, $G_{v\sigma}^*$, $G_{s\omega}^*$ and $G_{v\omega}^*$, in their μ dependences. Our results indicate that

eight-quark interactions are very important for the phase transition and must be studied in detail.

It is also very interesting to study effects of the interactions on color superconductivity itself and its correlation with the chiral phase transition. The study is now in progress.

Acknowledgments

The authors thank M. Tachibana for useful discussions and suggestions. H.K. also thanks T. Kunihiro for useful discussions and suggestions.

-
- [1] T. D. Lee, Nucl. Phys. **A750**, 1 (2005).
 - [2] M. Gyulassy and L. McLerran, Nucl. Phys. **A750**, 30 (2005).
 - [3] E. Shuryak, Nucl. Phys. **A750**, 64 (2005).
 - [4] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92**, 012001 (2004).
 - [5] K. Ghoroku, T. Sakaguchi, N. Uekusa, and M. Yahiro, Phys. Rev. D **71**, 106002 (2005).
 - [6] J. Kogut, M. Stone, H. W. Wyld, W. R. Gibbs, J. Shigemitsu, S. H. Shenker, and D. K. Sinclair, Phys. Rev. Lett. **50**, 393 (1983).
 - [7] Z. Fodor and S. D. Katz, J. High Energy Phys. **03**, 014 (2002); Prog. Theor. Phys. Suppl. **153**, 86 (2004).
 - [8] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); Phys. Rev. **124**, 246 (1961).
 - [9] S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992).
 - [10] T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994).
 - [11] M. Asakawa and K. Yazaki, Nucl. Phys. **A504**, 668 (1989).
 - [12] M. Buballa, Nucl. Phys. **A611**, 393 (1996).
 - [13] J. Berges and K. Rajagopal, Nucl. Phys. **B538**, 215 (1999).
 - [14] O. Scavenius, Á. Mócsy, I. N. Mishustin, and D. H. Rischke, Phys. Rev. C **64**, 045202 (2001).
 - [15] H. Fujii, Phys. Rev. D **67**, 094018 (2003).
 - [16] M. Kitazawa, T. Koide, T. Kunihiro, and Y. Nemoto, Prog. Theor. Phys. **108**, 929 (2002).
 - [17] A. A. Osipov, B. Hiller, and J. da Providência, Phys. Lett. B **634**, 48 (2006).
 - [18] A. A. Osipov, B. Hiller, J. Moreira, and A. H. Blin, Eur. Phys. J. C **46**, 225 (2006).

- [19] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
- [20] K.-I. Aoki, K. Morikawa, J.-I Sumi, H. Terao, and M. Tomoyose, Phys. Rev. D **61**, 045008 (2000).
- [21] O. Kiriya, M. Maruyama, and F. Takagi, Phys. Rev. D **63**, 116009 (2001).
- [22] Y. Hashimoto, Y. Tsue, and H. Fujii, Prog. Theor. Phys. **114**, 595 (2005).
- [23] W. Bentz, T. Horikawa, N. Ishii, and A. W. Thomas, Nucl. Phys. A **720**, 95 (2003).
- [24] W. Detmold, G. A. Miller, and J. R. Smith, Phys. Rev. C **73**, 015204 (2006).
- [25] J. D. Walecka, Ann. Phys. **83**, 491 (1974).
- [26] T. Kashiwa and T. Sakaguchi, Phys. Rev. D **68**, 065002 (2003).
- [27] T. Sakaguchi, M. Matsuzaki, H. Kouno, and M. Yahiro, arXiv:hep-ph/0606219 (2006).
- [28] H. Kouno, T. Sakaguchi, K. Kashiwa, M. Hamada, H. Tokudome, M. Matsuzaki, and M. Yahiro, Soryushiron Kenkyu **112** C67 (arXiv:nucl-th/0509057) (2005).
- [29] K. Tuchitani, H. Kouno, A. Hasegawa, and M. Nakano, Int. J. Mod. Phys. E **10**, 245 (2001).
- [30] H. Kouno, Y. Horinouchi, and K. Tuchitani, Prog. Theor. Phys. **112**, 831 (2004).
- [31] K. Tuchitani, Y. Horinouchi, K.-I. Makino, N. Noda, H. Kouno, T. Iwamitsu, M. Nakano, and A. Hasegawa, Int. J. Mod. Phys. E **14**, 955 (2005).
- [32] D. Zschesche, G. Zeeb, S. Schramm and H. Stöcker, J. Phys. G: Nucl. Part. Phys. **31**, 935 (2005).
- [33] D. Zschesche, G. Zeeb, and S. Schramm, arXiv:nucl-th/0602073 (2006).