Bulletin of Fukuoka University of Education. Vol. 48, Part III, 41–56 (1999)

On the Stability of a New Multistep Method

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(Received September 10, 1998)

ABSTRACT

A new multistep method for five-node through nine-node formulas for the minimum truncation error was derived in the work of Tamari, Furuki, Xu, and Yanagiwara (1998). For this new multistep method, the stability using the method of Yanagiwara (1995) may be expected.

At seven-node formulas, as the main corrector y_{n+2} was used in Tamari, Furuki, Xu, and Yanagiwara (1998), and y_{n+1} in Inamasu, Kaneko, and Yanagiwara (1994), though at the case of five-node formulas, the same formula was used in both papers.

So, in this paper the stability of our new multistep method will be considered for the four cases including seven-node formula, namely six-node through nine-node formulas.

1. INTRODUCTION

Attaching great importance to the minimum truncation error, in the work of Tamari, Furuki, Xu, and Yanagiwara (1998) derived a new multistep method for five-node through nine-node formulas. For our new multistep method it may be necessary to consider the stability, by the same method used in Yanagiwara (1995).

At seven-node formula, the main corrector was y_{n+2} in Tamari, Furuki, Xu, and Yanagiwara (1998), and y_{n+1} in Inamasu, Kaneko, and Yanagiwara (1994), though at the case of five-node formula, we used the same formula in both papers.

So, in this paper we will consider the stability of our new multistep method for four cases including seven-node formula, namely six-node through nine-node formulas.

2. THE STABILITY POLYNOMIAL OF SIX-NODE FORMULAS

In order to get the stability polynomial of our six-node formula, we rewrite our corrector formulas in Tamari, Furuki, Xu, and Yanagiwara (1998).

The correctors:

$$y_{n-2}=y_n-\frac{h}{90}(28y'_{n-2}+129y'_{n-1}+14y'_n$$

$$+14y'_{n+1}-6y'_{n+2}+y'_{n+3})+\frac{37}{3780}h^7y^{(7)}, \tag{2.1}$$

$$y_{n-1}=y_n-\frac{h}{1440}(-27y'_{n-2}+637y'_{n-1}$$

$$+1022y'_n-258y'_{n+1}+77y'_{n+2}-11y'_{n+3})-\frac{271}{60480}h^7y^{(7)}, \quad (2.2)$$

$$y_{n+1}=y_n+\frac{h}{1440}(11y'_{n-2}-93y'_{n-1}+802y'_n$$

$$+802y'_{n+1}-93y'_{n+2}+11y'_{n+3})-\frac{191}{60480}h^7y^{(7)}, \quad (2.3)$$

$$y_{n+2}=y_n+\frac{h}{90}(-y'_{n-1}+34y'_n+114y'_{n+1}+34y'_{n+2}-y'_{n+3})-\frac{1}{756}h^7y^{(7)}, \quad (2.4)$$

$$y_{n+3}=y_n+\frac{h}{160}(3y'_{n-2}-21y'_{n-1}+114y'_n$$

$$+114y'_{n+1}+219y'_{n+2}+51y'_{n+3})-\frac{29}{2240}h^7y^{(7)}, \quad (2.5)$$

Here, for the following differential equation:

$$y' = \lambda y, \quad (2.6)$$

when we fix up x_{n+1}, y_{n+1}, y'_{n+1}; x_n, y_n, y'_n; x_{n-1}, y_{n-1}, y'_{n-1}; $\cdots$, if y_{n+2} and y'_{n+2} converge by (2.4), i.e.,

$$y_{n+2}=y_n+\frac{h}{90}(-y'_{n-1}+34y'_n+114y'_{n+1}+34y'_{n+2}-y'_{n+3})-\frac{1}{756}h^7y^{(7)},$$

and if y_{n+3} and y'_{n+3} are arranged by (2.5), i.e.,

$$y_{n+3}=y_n+\frac{h}{160}(3y'_{n-2}-21y'_{n-1}+114y'_n+114y'_{n+1}+219y'_{n+2}+51y'_{n+3})$$

$$-\frac{29}{2240}h^7y^{(7)},$$

we have the following equations:

$$Hy_{n+3}+(90-34H)y_{n+2}$$

$$=114Hy_{n+1}+(90+34H)y_n-Hy_{n-1}, \quad (2.7)$$

$$(160-51H)y_{n+3}-219Hy_{n+2}$$

$$=114Hy_{n+1}+(160+114H)y_n-21Hy_{n-1}+3Hy_{n-2}, \quad (2.8)$$

where $H=\lambda h$.

Solving (2.7) and (2.8), we get the following equation:

$$(1953H^2-10030H+14400)y_{n+2}=(18240-5928H)Hy_{n+1}$$

$$-(1848H^2-690H-14400)y_n+(72\,H-160)Hy_{n-1}-3H^2y_{n-2}. \tag{2.9}$$

Therefore, we obtain the following stability polynomial:

$$(1953H^2-10030H+14400)\mu^4+(5928H-18240)H\mu^3$$

$$+(1848H^2-690H-14400)\mu^2-(72H-160)H\mu+3H^2=0. \tag{2.10}$$

3. A STUDY OF THE EQUATION (2.10)

To get the domain: $|\mu|<1$, setting $\mu=e^{i\theta}$, we have the following equation:

$$(1953H^2-10030H+14400)e^{4i\theta}+(5928H-18240)He3^{3i\theta}$$

$$+(1848H^2-690H-14400)e^{2i\theta}-(72H-160)He^{i\theta}+3H^2=0. \tag{3.1}$$

Arranging this by H, we get the next equation:

$$3(651e^{4i\theta}+1976e^{3i\theta}+616e^{2i\theta}-24e^{i\theta}+1)H^2$$

$$-10(1003e^{4i\theta}+1824e^{3i\theta}+69e^{2i\theta}-16e^{i\theta})H$$

$$+14400(e^{2i\theta}-1)e^{2i\theta}=0. \tag{3.2}$$

Solving (3.2) by H, we obtain the following equation:

$$H=\frac{-f1\pm\sqrt{f1\times f1-4\times f0\times f2}}{2\times f0}, \tag{3.3}$$

where $f0$, $f1$, and $f2$ are the coefficients of H^2, H, and H^0 respectively.

We change θ from $-\pi$ to π, then we have the following **Figure 1**.

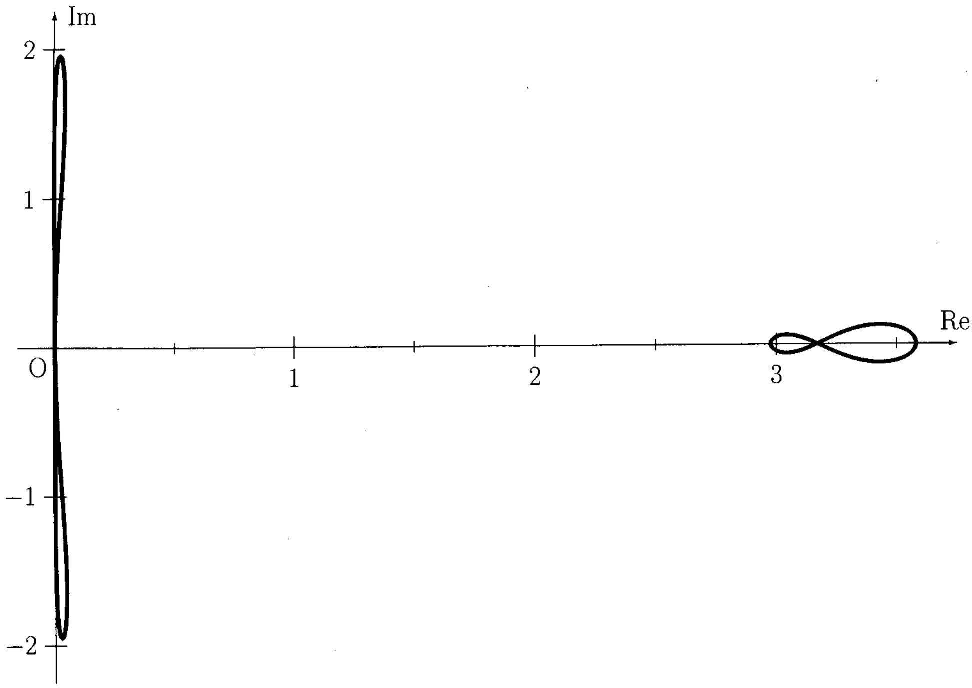

Figure 1

By the **Figure 1**, we see that our corrector formula (2.4) is not **"absolutely stable"**. But, (2.4) is **"zero stable"**, by the same reason as in Inamasu, Kaneko, and Yanagiwara (1994).

4. A STABILITY POLYNOMIAL OF SEVEN-NODE FORMULAS

In order to get the stability polynomial of our seven-node formulas, we rewrite our corrector formulas in Tamari, Furuki, Xu, and Yanagiwara (1998).

The correctors:

$$y_{n-3}=y_n-\frac{h}{2240}(685y'_{n-3}+3240y'_{n-2}+1161y'_{n-1}+2176y'_n$$
$$-729y'_{n+1}+216y'_{n+2}-29y'_{n+3})-\frac{9}{896}h^8y^{(8)}, \quad (4.1)$$

$$y_{n-2}=y_n-\frac{h}{3780}(-37y'_{n-3}+1398y'_{n-2}+4863y'_{n-1}+1328y'_n$$
$$+33y'_{n+1}-30y'_{n+2}+5y'_{n+3})+\frac{1}{756}h^8y^{(8)}, \quad (4.2)$$

$$y_{n-1}=y_n-\frac{h}{60480}(271y'_{n-3}-2760y'_{n-2}+30819y'_{n-1}+37504y'_n$$

$$-6771y'_{n+1}+1608y'_{n+2}-191y'_{n+3})-\frac{191}{120960}h^8y^{(8)}, \tag{4.3}$$

$$y_{n+1}=y_n+\frac{h}{60480}(-191y'_{n-3}+1608y'_{n-2}-6771y'_{n-1}+37504y'_n$$

$$+30819y'_{n+1}-2760y'_{n+2}+271y'_{n+3})-\frac{191}{120960}h^8y^{(8)}, \tag{4.4}$$

$$y_{n+2}=y_n+\frac{h}{3780}(5y'_{n-3}-30y'_{n-2}+33y'_{n-1}+1328y'_n$$

$$+4863y'_{n+1}+1398y'_{n+2}-37y'_{n+3})+\frac{1}{756}h^8y^{(8)}, \tag{4.5}$$

$$y_{n+3}=y_n+\frac{h}{2240}(-29y'_{n-3}+216y'_{n-2}-729y'_{n-1}+2176y'_n$$

$$+1161y'_{n+1}+3240y'_{n+2}+685y'_{n+3})-\frac{9}{896}h^8y^{(8)}, \tag{4.6}$$

Here, for the above differential equation (2.6), i.e.,

$$y' = \lambda y,$$

when we fix up x_{n+1}, y_{n+1}, y'_{n+1}; x_n, y_n, y'_n; x_{n-1}, y_{n-1}, y'_{n-1}; $\cdots$, if y_{n+2} and y'_{n+2} converge by (4.5), i.e.,

$$y_{n+2}=y_n+\frac{h}{3780}(5y'_{n-3}-30y'_{n-2}+33y'_{n-1}+1328y'_n$$

$$+4863y'_{n+1}+1398y'_{n+2}-37y'_{n+3})+\frac{1}{756}h^8y^{(8)},$$

and if y_{n+3} and y'_{n+3} are arranged by (4.6), i.e.,

$$y_{n+3}=y_n+\frac{h}{2240}(-29y'_{n-3}+216y'_{n-2}-729y'_{n-1}+2176y'_n$$

$$+1161y'_{n+1}+3240y'_{n+2}+685y'_{n+3})-\frac{9}{896}h^8y^{(8)},$$

we have the following equations:

$$37Hy_{n+3}+(3780-1398H)y_{n+2}=4863Hy_{n+1}$$

$$+(3780+1328H)y_n+33Hy_{n-1}-30Hy_{n-2}+5Hy_{n-3}, \tag{4.7}$$

$$(2240-685H)y_{n+3}-3240Hy_{n+2}=1161Hy_{n+1}$$

$$+(2240+2176H)y_n-729Hy_{n-1}+216Hy_{n-2}-29Hy_{n-3}, \tag{4.8}$$

where $H=\lambda h$.

Solving (4.7) and (4.8), we get the following equation:

$$(76965H^2-408630H+604800)y_{n+2}=(778080-241008H)Hy_{n+1}$$
$$-(70728H^2-21610H-604800)y_n+(312H+5280)Hy_{n-1}$$
$$+(897H-4800)Hy_{n-2}-(168H-800)Hy_{n-3}. \tag{4.9}$$

Therefore, we obtain the following stability polynomial:

$$(76965H^2-408630H+604800)\mu^5-(778080-241008H)H\mu^4$$
$$+(70728H^2-21610H-604800)\mu^3-(312H+5280)H\mu^2$$
$$-(897H-4800)H\mu+(168H-800)H=0. \tag{4.10}$$

5. A STUDY OF THE EQUATION (4.10)

To get the domain: $|\mu|<1$, setting $\mu=e^{i\theta}$, we have the following equation:

$$(76965H^2-408630H+604800)e^{5i\theta}+(241008H-778080)He^{4i\theta}$$
$$+(70728H^2-21610H-604800)e^{3i\theta}-(312H+5280)He^{2i\theta}$$
$$-(897H-4800)He^{i\theta}+168H^2-800H=0. \tag{5.1}$$

Arranging this by H, we get the next equation:

$$(76965e^{5i\theta}+241008e^{4i\theta}+70728e^{3i\theta}-312e^{2i\theta}-897e^{i\theta}+168)H^2$$
$$-10(40863e^{5i\theta}+77808e^{4i\theta}+2161e^{3i\theta}+528e^{2i\theta}-480e^{i\theta}+80)H$$
$$+604800(e^{2i\theta}-1)e^{3i\theta}=0. \tag{5.2}$$

Solving (5.2) by H, we change θ from $-\pi$ to π then we have the following **Figure 2**.

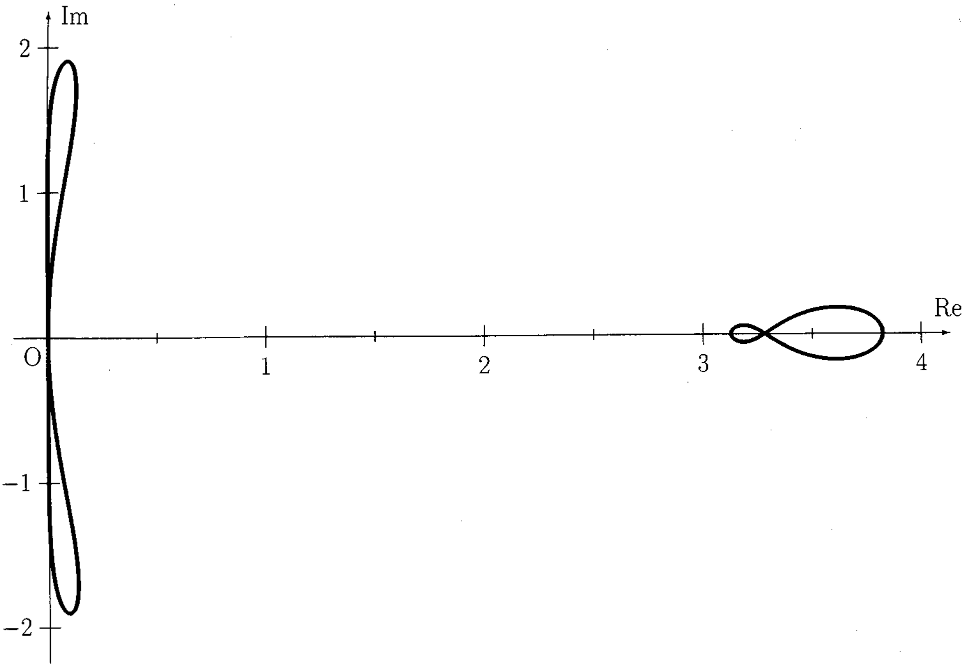

Figure 2

By the **Figure 2**, we see that our corrector formula (4.5) is not **"absolutely stable"**. But, (4.5) is **"zero stable"**, by the same reason as in Inamasu, Kaneko, and Yanagiwara (1994).

6. THE STABILITY POLYNOMIAL OF EIGHT-NODE FORMULAS

In order to get the stability polynomial of our eight-node formulas, we rewrite our corrector formulas used in Tamari, Furuki, Xu, and Yanagiwara (1998).

The correctors:

$$y_{n-3}=y_n-\frac{h}{4480}(1325y'_{n-3}+6795y'_{n-2}+1377y'_{n-1}+5927y'_n-3033y'_{n+1}$$
$$+1377y'_{n+2}-373y'_{n+3}+45y'_{n+4})+\frac{369}{44800}h^9y^{(9)}, \tag{6.1}$$

$$y_{n-2}=y_n-\frac{h}{3780}(-32y'_{n-3}+1363y'_{n-2}+4968y'_{n-1}+1153y'_n+208y'_{n+1}$$
$$-135y'_{n+2}+40y'_{n+3}-5y'_{n+4})-\frac{127}{113400}h^9y^{(9)}, \tag{6.2}$$

$$y_{n-1}=y_n-\frac{h}{120960}(351y'_{n-3}-4183y'_{n-2}+57627y'_{n-1}+81693y'_n-20227y'_{n+1}$$
$$+7227y'_{n+2}-1719y'_{n+3}+191y'_{n+4})+\frac{3233}{3628800}h^9y^{(9)}, \tag{6.3}$$

$$y_{n+1}=y_n+\frac{h}{120960}(-191y'_{n-3}+1879y'_{n-2}-9531y'_{n-1}+68323y'_n+68323y'_{n+1}$$
$$-9531y'_{n+2}+1879y'_{n+3}-191y'_{n+4})+\frac{2497}{3628800}h^9y^{(9)}, \tag{6.4}$$

$$y_{n+2}=y_n+\frac{h}{3780}(5y'_{n-2}-72y'_{n-1}+1503y'_n+4688y'_{n+1}$$
$$+1503y'_{n+2}-72y'_{n+3}+5y'_{n+4})-\frac{23}{113400}h^9y^{(9)}, \tag{6.5}$$

$$y_{n+3}=y_n+\frac{h}{4480}(-13y'_{n-3}+117y'_{n-2}-513y'_{n-1}+2777y'_n$$
$$+3897y'_{n+1}+5535y'_{n+2}+1685y'_{n+3}-45y'_{n+4})$$
$$+\frac{81}{44800}h^9y^{(9)}, \tag{6.6}$$

$$y_{n+4}=y_n+\frac{h}{945}(8y'_{n-3}-64y'_{n-2}+216y'_{n-1}-106y'_n$$
$$+1784y'_{n+1}+216y'_{n+2}+1448y'_{n+3}+278y'_{n+4}$$
$$-\frac{107}{14175}h^9y^{(9)}, \tag{6.7}$$

Here, for the above differential equation (2.6), i.e.,

$$y' = \lambda y,$$

when we fix up x_{n+1}, y_{n+1}, y'_{n+1}; x_n, y_n, y'_n; x_{n-1}, y_{n-1}, y'_{n-1}; $\cdots$, if y_{n+2} and y'_{n+2} converge by (6.5), i.e.,

$$y_{n+2}=y_n+\frac{h}{3780}(5y'_{n-2}-72y'_{n-1}+1503y'_n$$
$$+4688y'_{n+1}+1503y'_{n+2}-72y'_{n+3}+5y'_{n+4})-\frac{23}{113400}h^9y^{(9)},$$

if y_{n+3} and y'_{n+3} are arranged by (6.6), i.e.,

$$y_{n+3}=y_n+\frac{h}{4480}(-13y'_{n-3}+117y'_{n-2}-513y'_{n-1}+2777y'_n$$
$$+3897y'_{n+1}+5535y'_{n+2}+1685y'_{n+3}-45y'_{n+4})+\frac{81}{44800}h^9y^{(9)},$$

and if y_{n+4} and y'_{n+4} are arranged by (6.7), i.e.,

$$y_{n+4}=y_n+\frac{h}{945}(8y'_{n-3}-64y'_{n-2}+216y'_{n-1}-106y'_n$$

$$+1784y'_{n+1}+216y'_{n+2}+1448y'_{n+3}+278y'_{n+4})-\frac{107}{14175}h^9y^{(9)},$$

we have the following equations:

$$-5Hy_{n+4}+72Hy_{n+3}+(3780-1503H)y_{n+2}$$

$$=4688Hy_{n+1}+(3780+1503H)y_n-72Hy_{n-1}+5Hy_{n-2}, \tag{6.8}$$

$$45Hy_{n+4}+(4480-1685H)y_{n+3}-5535Hy_{n+2}=3897Hy_{n+1}$$

$$+(4480+2777H)y_n-513Hy_{n-1}+117Hy_{n-2}-13Hy_{n-3}, \tag{6.9}$$

$$(945-278H)y_{n+4}-1448Hy_{n+3}-216Hy_{n+2}=1784Hy_{n+1}$$

$$+(945-106H)y_n+216Hy_{n-1}-64Hy_{n-2}+8Hy_{n-3}, \tag{6.10}$$

where $H=\lambda h$.

Solving (6.8), (6.9), and (6. 10), we get the following equation:

$$(135961110H^3-950556285H^2+2441407500H-2286144000)y_{n+2}$$

$$=-(371205216H^2-1932661800H+2835302400)Hy_{n+1}$$

$$-(125460756H^3-331757860H^2-663900300H+2286144000)y_n$$

$$+(7645824H^2-34866000H+43545600)Hy_{n-1}$$

$$-(884106H^2-3369015H+3024000)Hy_{n-2}$$

$$+(56544H-151960)H^2y_{n-3}. \tag{6.11}$$

Therefore, we obtain the following stability polynomial:

$$(135961110H^3-950556285H^2+2441407500H-2286144000)\mu^5$$

$$+(371205216H^2-1932661800H+2835302400)H\mu^4$$

$$+(125460756H^3-331757860H^2-663900300H+2286144000)\mu^3$$

$$-(7645824H^2-34866000H+43545600)H\mu^2$$

$$+(884106H^2-3369015H+3024000)H\mu$$

$$-(56544H-151960)H^2=0. \qquad (6.12)$$

7. A STUDY OF THE EQUATION (6.12)

To get the domain : $|\mu|<1$, setting $\mu = e^{i\theta}$, we have the following equation :

$$(135961110H^3-950556285H^2+2441407500H-2286144000)e^{5i\theta}$$

$$+(371205216H^2-1932661800H+2835302400)He^{4i\theta}$$

$$+(125460756H^3-331757860H^2-663900300H+2286144000)e^{3i\theta}$$

$$-(7645824H^2-34866000H+43545600)He^{2i\theta}$$

$$+(884106H^2-3369015H+3024000)He^{i\theta}+168H^2$$

$$-(56544H-151960)H^2=0. \qquad (7.1)$$

Arranging this by H, we get the next equation :

$$(135961110e^{5i\theta}+371205216e^{4i\theta}+125460756e^{3i\theta}$$

$$-7645824e^{2i\theta}+884106e^{i\theta}-56544)H^3$$

$$-(950556286e^{5i\theta}+1932661800e^{4i\theta}+331757860e^{3i\theta}$$

$$-34866000e^{2i\theta}+3369015e^{i\theta}-151960)H^2$$

$$+100(24414075e^{5i\theta}+28353024e^{4i\theta}-6639003e^{3i\theta}$$

$$-435456e^{2i\theta}+30240e^{i\theta})H$$

$$-2286144000(e^{2i\theta}-1)e^{3i\theta}=0. \qquad (7.2)$$

Solving (7.2) by H, using the Cardan's method, we change θ from $-\pi$ to π, then we have the following **Figure 3**.

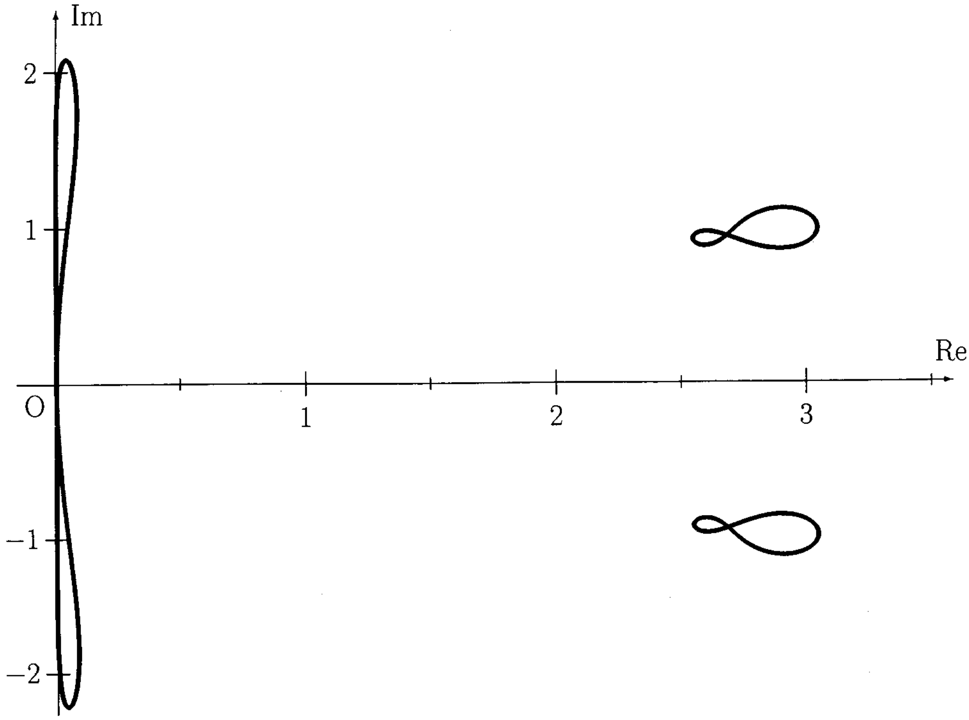

Figure 3

By the **Figure 3**, we see that our corrector formula (6.5) is not **"absolutely stable"**. But, (6.5) is **"zero stable"**, by the same reason as in Inamasu, Kaneko, and Yanagiwara (1944).

8. THE STABILITY POLYNOMIAL OF NINE-NODE FORMULAS

In order to get the stability polynomial of our nine-node formulas, we rewrite our corrector formulas in Tamari, Furuki, Xu, and Yanagiwara (1998).

The correctors:

$$y_{n-4}=y_n-\frac{h}{14175}(4063y'_{n-4}+22576y'_{n-3}+244y'_{n-2}+32752y'_{n-1}-9080y'_n$$
$$+9232y'_{n+1}-3956y'_{n+2}+976y'_{n+3}-107y'_{n+4})-\frac{94}{14175}h^{10}y^{(10)}, \quad (8.1)$$

$$y_{n-3}=y_n-\frac{h}{44800}(-369y'_{n-4}+16202y'_{n-3}+57618y'_{n-2}+34434y'_{n-1}+33440y'_n$$
$$-9666y'_{n+1}+3438y'_{n+2}-778y'_{n+3}+81y'_{n+4})+\frac{113}{89600}h^{10}y^{(10)}, \quad (8.2)$$

$$y_{n-2}=y_n-\frac{h}{113400}(127y'_{n-4}-1976y'_{n-3}+44446y'_{n-2}+141928y'_{n-1}+43480y'_n$$

$$-872y'_{n+1}-494y'_{n+2}+184y'_{n+3}-23y'_{n+4})-\frac{23}{113400}h^{10}y^{(10)}, \quad (8.3)$$

$$y_{n-1}=y_n-\frac{h}{3628800}(-3233y'_{n-4}+36394y'_{n-3}-216014y'_{n-2}+1909858y'_{n-1}$$

$$+2224480y'_n-425762y'_{n+1}+126286y'_{n+2}-25706y'_{n+3}+2497y'_{n+4})$$

$$+\frac{2497}{7257600}h^{10}y^{(10)}, \quad (8.4)$$

$$y_{n+1}=y_n+\frac{h}{3628800}(2497y'_{n-4}-25706y'_{n-3}+126286y'_{n-2}-425762y'_{n-1}$$

$$+2224480y'_n+1909858y'_{n+1}-216014y'_{n+2}+36394y'_{n+3}-3233y'_{n+4})$$

$$+\frac{2497}{7257600}h^{10}y^{(10)}, \quad (8.5)$$

$$y_{n+2}=y_n+\frac{h}{113400}(-23y'_{n-4}+184y'_{n-3}-494y'_{n-2}-872y'_{n-1}+43480y'_n$$

$$+141928y'_{n+1}+4446y'_{n+2}-1976y'_{n+3}+127y'_{n+4})-\frac{23}{113400}h^{10}y^{(10)}, \quad (8.6)$$

$$y_{n+3}=y_n+\frac{h}{44800}(81y'_{n-4}-778y'_{n-3}+3438y'_{n-2}-9666y'_{n-1}+33440y'_n$$

$$+34434y'_{n+1}+57618y'_{n+2}+16202y'_{n+3}-369y'_{n+4})+\frac{113}{89600}h^{10}y^{(10)}, \quad (8.7)$$

$$y_{n+4}=y_n+\frac{h}{14175}(-107y'_{n-4}+976y'_{n-3}-3956y'_{n-2}+9232y'_{n-1}-9080y'_n$$

$$+32752y'_{n+1}+244y'_{n+2}+22576y'_{n+3}+4063y'_{n+4})-\frac{94}{14175}h^{10}y^{(10)}, \quad (8.8)$$

Here, for the above differential equation (2.6), i.e.,

$$y' = \lambda y,$$

when we fix up x_{n+1}, y_{n+1}, y'_{n+1}; x_n, y_n, y'_n; x_{n-1}, y_{n-1}, y'_{n-1}; $\cdots$, if y_{n+2} and y'_{n+2} converge by (8.6), i.e.,

$$y_{n+2}=y_n+\frac{h}{113400}(-23y'_{n-4}+184y'_{n-3}-494y'_{n-2}-872y'_{n-1}+43480y'_n$$

$$+141928y'_{n+1}+44446y'_{n+2}-1976y'_{n+3}+127y'_{n+4})-\frac{23}{113400}h^{10}y^{(10)},$$

if y_{n+3} and y'_{n+3} are arranged by (8.7), i.e.,

$$y_{n+3}=y_n+\frac{h}{44800}(81y'_{n-4}-778y'_{n-3}+3438y'_{n-2}-9666y'_{n-1}+33440y'_n$$

$$+34434y'_{n+1}+57618y'_{n+2}+16202y'_{n+3}-369y'_{n+4})+\frac{113}{89600}h^{10}y^{(10)},$$

and if y_{n+4} and y'_{n+4} are arranged by (8.8), i.e.,

$$y_{n+4}=y_n+\frac{h}{14175}(-107y'_{n-4}+976y'_{n-3}-3956y'_{n-2}+9232y'_{n-1}-9080y'_n$$

$$+32752y'_{n+1}+244y'_{n+2}+22576y'_{n+3}+4063y'_{n+4})-\frac{94}{14175}h^{10}y^{(10)},$$

we have the following equations:

$$-127Hy_{n+4}+1976Hy_{n+3}+(113400-44446H)y_{n+2}$$

$$=141928Hy_{n+1}+(113400+43480H)y_n-872Hy_{n-1}-494Hy_{n-2}$$

$$+184Hy_{n-3}-23Hy_{n-4}, \tag{8.9}$$

$$369Hy_{n+4}+(44800-16202H)y_{n+3}-57618Hy_{n+2}$$

$$=34434Hy_{n+1}+(44800+33440H)y_n-9666Hy_{n-1}+3438Hy_{n-2}$$

$$-778Hy_{n-3}+81Hy_{n-4}, \tag{8.10}$$

$$(14175-4063H)y_{n+4}-22576Hy_{n+3}-244Hy_{n+2}$$

$$=32752Hy_{n+1}+(14175-9080H)y_n+9232Hy_{n-1}-3956Hy_{n-2}$$

$$+976Hy_{n-3}-107Hy_{n-4}, \tag{8.11}$$

where $H = \lambda h$.

Solving (8.9), (8.10), and (8.11), we get the following equation:

$$(26156958290H^3-188799509710H^2+499401012600H-480090240000)y_{n+2}$$

$$=-(72379639872H^2-394719192000H+600866380800)Hy_{n+1}$$

$$-(24005720880H^3-66729676195H^2-134985425400H+480090240000)y_n$$

$$+(1214992960H^2-4548393920H+3691699200)Hy_{n-1}$$

$$-(40534110H^2+563780070H-2091398400)Hy_{n-2}$$

$$- (25812480H^2 - 322703360H + 778982400)Hy_{n-3}$$

$$+ (4539772H^2 - 43941275H + 97372800)Hy_{n-4}. \tag{8.12}$$

Therefore, we obtain the following stability polynomial:

$$(26156958290H^3 - 188799509710H^2 + 499401012600H - 480090240000)\mu^6$$

$$+ (72379639872H^2 - 394719192000H + 600866380800)H\mu^5$$

$$+ (24005720880H^3 - 66729676195H^2 - 134985425400H + 480090240000)\mu^4$$

$$- (1214992960H^2 - 4548393920H + 3691699200)H\mu^3$$

$$+ (40534110H^2 + 563780070H - 2091398400)H\mu^2$$

$$+ (25812480H^2 - 322703360H + 778982400)H\mu$$

$$-4539772H^3 + 43941275H^2 - 97372800H = 0. \tag{8.13}$$

9. A STUDY OF THE EQUATION (8.13)

To get the domain: $|\mu| < 1$, setting $\mu = e^{i\theta}$, we have the following equation:

$$(26156958290H^3 - 188799509710H^2 + 499401012600H - 480090240000)e^{6i\theta}$$

$$+ (72379639872H^2 - 394719192000H + 600866380800)He^{5i\theta}$$

$$+ (24005720880H^3 - 66729676195H^2 - 134985425400H + 480090240000)e^{4i\theta}$$

$$- (1214992960H^2 - 4548393920H + 3691699200)He^{3i\theta}$$

$$+ (40534110H^2 + 563780070H - 2091398400)He^{2i\theta}$$

$$+ (25812480H^2 - 322703360H + 778982400)He^{i\theta}$$

$$-4539772H^3 + 43941275H^2 - 97372800H = 0. \tag{9.1}$$

Arranging this by H, we get the next equation:

$$(26156958290e^{6i\theta} + 72379639872e^{5i\theta} + 24005720880e^{4i\theta}$$

$$-1214992960e^{3i\theta}+40534110e^{2i\theta}+25812480e^{i\theta}-4539772)H^3$$

$$-(188799509710e^{6i\theta}+394719192000e^{5i\theta}+667296761 95e^{4i\theta}$$

$$-4548393920e^{3i\theta}-563780070e^{2i\theta}+322703360e^{i\theta}-43941275)H^2$$

$$+100(4994010126e^{6i\theta}+6008663808e^{5i\theta}-1349854254e^{4i\theta}$$

$$-36916992e^{3i\theta}-20913984e^{2i\theta}+7789824e^{i\theta}-973728)H$$

$$-480090240000(e^{2i\theta}-1)e^{4i\theta}=0. \qquad (9.2)$$

Solving (9.2) by H, using the Cardan's method, we change θ from $-\pi$ to π, then we have the following **Figure 4.**

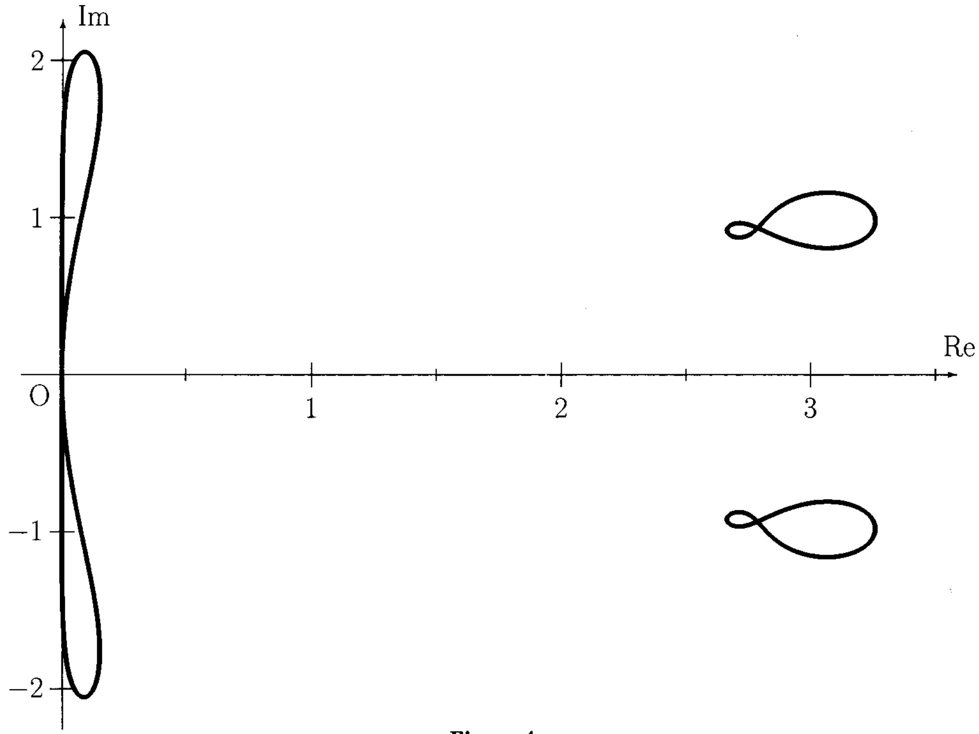

Figure 4

By the **Figure 4**, we see that our corrector formula (8.6) is not **"absolutely stable"**. But, (8.6) is **"zero stable"**, by the same reason as in Inamasu, Kaneko, and Yanagiwara (1944).

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