

On an Interpolation Using Taylor Expansion

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Abstract

In [3] and [4], we obtained the ten-node interpolation and the twelve-node interpolation using Taylor expansion, respectively. In the present paper, we shall show interpolation formulas from two-node to thirteen-node excepting ten-node and twelve-node, using Taylor expansion by the same method in [3] and [4].

1. Introduction

In Section 2, we show two-node interpolation formula. Section 3 employs three-node formula. In Section k ($4 \leq k \leq 9$), we study the k -node interpolation. In Section 10, we want study the eleven-node formula. And in Section 11, we show the thirteen-node formula. In Section 12, when we solve numerically a differential equation, we use several interpolation formulas by halving the step size. In Tables 1, 3 and 5, if y_n does not converge after at most four times iteration, we halve the step size and then we compare the degree of precision of numerical solution for a differential equation. In Section 13, we show the thirteen-node formula. In [3] and [4], we showed the relation between $k_{-(k-i)}^{+i}$ and $k_{-i}^{+(k-i)}$.

2. Two-node interpolation

From $c_1 f(h/2) + c_2 f(-h/2)$, we obtain the following equation.

$$(c_1 + c_2)f(0) = c_1 f(h/2) + c_2 f(-h/2) - (c_1 - c_2)h/2 f'(0) - (c_1 + c_2)(h/2)^2 f''(0)/2 + \dots$$

Here, setting $c_1 + c_2 = 1$ and $c_1 - c_2 = 0$, we have $c_1 = c_2 = 1/2$. Using these values, we get the following formula :

$$2_{-1}^{+1} : f(0) = (f(h/2) + f(-h/2))/2 - h^2 f''(\xi)/8. \quad (2.1)$$

3. Three-node interpolation

(1). 3_{-2}^{+1} formula.

From $c_1 f(h/2) + c_2 f(-h/2) + c_3 f(-3h/2)$, we have the following equation.

$$(c_1 + c_2 + c_3)f(0) = c_1 f(h/2) + c_2 f(-h/2) + c_3 f(-3h/2) - (c_1 - c_2 - 3c_3)h/2 f'(0) \\ - (c_1 + c_2 + 3^2 c_3)(h/2)^2 f''(0)/2 - (c_1 - c_2 - 3^3 c_3)(h/2)^3 f^{(3)}(0)/6 + \dots.$$

Here, setting $c_1 + c_2 + c_3 = 1$, $c_1 - c_2 - 3c_3 = 0$, and $c_1 + c_2 + 3^2 c_3 = 0$, we get the following values :

$$c_1 = 3/8, \quad c_2 = 3/4, \quad c_3 = -1/8.$$

From these values, we obtain the following interpolation formula.

$$f(0) = (3f(h/2) + 6f(-h/2) - f(-3h/2))/8 - h^3 f^{(3)}(\xi)/16. \quad (3.1)$$

(2). 3_{-1}^{+2} formula.

In [3] and [4], excepting the case $(2k)_{-k}^{+k}$, we showed that when $1 \leq i \leq \frac{k}{2}$, the relations between the coefficient $c_j (1 \leq j \leq k)$ of the interpolation formula $k_{-(k-i)}^{+i}$:

$$f(0) = c_1 f(\frac{1}{2}h) + c_2 f(-\frac{1}{2}h) + \dots + c_k f(-\frac{2(k-i)-1}{2}h)$$

and the coefficient $d_j (1 \leq j \leq k)$ of the interpolation formula $k_{-i}^{+(k-i)}$:

$$f(0) = d_1 f(\frac{1}{2}h) + d_2 f(-\frac{1}{2}h) + \dots + d_k f(\frac{2(k-i)-1}{2}h)$$

are

$$c_1 = d_2, \quad c_2 = d_1, \quad \dots, \quad c_k = d_k$$

For example, in the following cases :

$$9_{-6}^{+3} : f(0) = c_1 f(h/2) + c_2 f(-h/2) + c_3 f(3h/2) + c_4 f(-3h/2) + c_5 f(5h/2) + c_6 f(-5h/2) \\ + c_7 f(-7h/2) + c_8 f(-9h/2) + c_9 f(-11h/2)$$

$$9_{-3}^{+6} : f(0) = d_1 f(h/2) + d_2 f(-h/2) + d_3 f(3h/2) + d_4 f(-3h/2) + d_5 f(5h/2) + c_6 f(-5h/2) \\ + d_7 f(7h/2) + c_8 f(9h/2) + c_9 f(11h/2),$$

we have

$$c_1 = d_2, \quad c_2 = d_1, \quad c_3 = d_4, \quad c_4 = d_3, \quad c_5 = d_6, \quad c_6 = d_5, \quad c_7 = d_7, \quad c_8 = d_8, \quad c_9 = d_9.$$

From $c_1 f(h/2) + c_2 f(-h/2) + c_3 f(3h/2)$, we obtain the following interpolation formula.

$$f(0) = (6f(h/2) + 3f(-h/2) - f(3h/2))/8 + h^3 f^{(3)}(\xi)/16. \quad (3.2)$$

4. Four-node interpolation

(1) 4_{-2}^{+2} formula.

From $c_1(f(h/2) + f(-h/2)) + c_2(f(3h/2) + f(-3h/2))$, we have the following equation :

$$2(c_1+c_2)f(0) = c_1(f(h/2)+f(-h/2))+c_2(f(3h/2)+f(-3h/2))-2(c_1+3^2c_2)(h/2)^2f''(0)/2 \\ - 2(c_1+3^4c_2)(h/2)^4f^{(4)}(0)/24+\dots.$$

Setting $2(c_1+c_2) = 1$, $c_1+9c_2 = 0$, we see $c_1 = 9/16$, $c_2 = -1/16$. So, we obtain the following formula :

$$f(0) = (9(f(h/2)+f(-h/2))-(f(3h/2)+f(-3h/2)))/16+3h^4f^{(4)}(\xi)/128. \quad (4.1)$$

(2) 4_{-3}^{+1} formula.

From $c_1f(h/2)+c_2f(-h/2)+c_3f(-3h)+c_4f(-5h)$, we get the following equation :

$$(c_1+c_2+c_3+c_4)f(0) = c_1f(h/2)+c_2f(-h/2)+c_3f(-3h/2)+c_4f(-5h/2)-(c_1-c_2-3c_3 \\ - 5c_4)(h/2)f'(0)-(c_1+c_2+3^2c_3+5^2c_4)(h/2)^2f''(0)/2-(c_1-c_2-3^3c_3 \\ - 5^3c_4)(h/2)^3f^{(3)}(0)/6-(c_1+c_2+3^4c_3+5^4c_4)(h/2)^4f^{(4)}(\xi)/24.$$

Setting $\sum_{k=1}^4 c_k = 1$, $c_1-c_2-3c_3-5c_4 = 0$, $c_1+c_2+3^2c_3+5^2c_4 = 0$, $c_1-c_2-3^3c_3-5^3c_4 = 0$, we have $c_1 = 5/16$, $c_2 = 15/16$, $c_3 = -5/16$, $c_4 = 1/16$. From these values, we obtain the following formula :

$$f(0) = (5f(h/2)+15f(-h/2)-5f(-3h/2)+f(-5h/2))/16-5h^4f^{(4)}(\xi)/128. \quad (4.2)$$

5. Five-node interpolation

(1) 5_{-3}^{+2} formula.

Solving the following simultaneous equation :

$$\sum_{k=1}^5 c_k = 1, \quad c_1-c_2+3(c_3-c_4)-5c_5 = 0, \quad c_1+c_2+3^2(c_3+c_4)+5^2c_5 = 0, \\ c_1-c_2+3^3(c_3-c_4)-5^3c_5 = 0, \quad c_1+c_2+3^4(c_3+c_4)+5^4c_5 = 0,$$

we get $c_1 = 60/128$, $c_2 = 90/128$, $c_3 = -5/128$, $c_4 = -20/128$, $c_5 = 3/128$.

From these values, we obtain the following 5_{-3}^{+2} formula :

$$f(0) = (60f(h/2)+90f(-h/2)-5f(3h/2)-20f(-3h/2)+3f(-5h/2))/128-3h^5f^{(5)}(\xi)/256. \quad (5.1)$$

(2) 5_{-4}^{+1} formula.

From the following simultaneous equation :

$$\sum_{k=1}^5 c_k = 1, \quad c_1-c_2-3c_3-5c_4-7c_5 = 0, \quad c_1+c_2+3^2c_3+5^2c_4+7^2c_5 = 0, \\ c_1-c_2-3^3c_3-5^3c_4-7^3c_5 = 0, \quad c_1+c_2+3^4c_3+5^4c_4+7^4c_5 = 0,$$

we have $c_1 = 35/128$, $c_2 = 140/128$, $c_3 = -70/128$, $c_4 = 28/128$, $c_5 = -5/128$.

From these values, we obtain the following formula :

$$f(0) = (35f(h/2)+140f(-h/2)-70f(-3h/2)+28f(-5h/2)-5f(-7h/2))/128-7h^5f^{(5)}(\xi)/256. \quad (5.2)$$

6. Six-node interpolation

(1) 6_{-3}^{+3} formula.

Solving the following equations :

$$\sum_{k=1}^3 c_k = 1/2, \quad c_1 + 3^2 c_2 + 5^2 c_3 = 0, \quad c_1 + 3^4 c_2 + 5^4 c_3 = 0,$$

we get $c_1 = 150/256$, $c_2 = -25/256$, $c_3 = 3/256$.

From these values, we obtain the following formula :

$$f(0) = (150(f(h/2) + f(-h/2)) - 25(f(3h/2) + f(-3h/2)) + 3(f(5h/2) + f(-5h/2)))/256 - 5h^6 f^{(6)}(\xi)/2048. \quad (6.1)$$

(2) 6_{-4}^{+2} formula.

From the following equations :

$$\begin{aligned} \sum_{k=1}^6 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) - 5c_5 - 7c_6 = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2 c_5 + 7^2 c_6 &= 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) - 5^3 c_5 - 7^3 c_6 &= 0, \\ c_1 + c_2 + 3^4(c_3 + c_4) + 5^4 c_5 + 7^4 c_6 &= 0, \\ c_1 - c_2 + 3^5(c_3 - c_4) - 5^5 c_5 - 7^5 c_6 &= 0, \end{aligned}$$

we get $c_1 = 105/256$, $c_2 = 210/256$, $c_3 = -7/256$, $c_4 = -70/256$, $c_5 = 21/256$, $c_6 = -3/256$.

From these values, we obtain the following formula :

$$f(0) = (105f(h/2) + 210f(-h/2) - 7f(3h/2) - 70f(-3h/2) + 21f(-5h/2) - 3f(-7h/2))/256 + 7h^6 f^{(6)}(\xi)/1024. \quad (6.2)$$

(3) 6_{-5}^{+1} formula.

From the following equations :

$$\begin{aligned} \sum_{k=1}^6 c_k &= 1, \quad c_1 - c_2 - 3c_3 - 5c_4 - 7c_5 - 9c_6 = 0, \\ c_1 + c_2 + 3^2 c_3 + 5^2 c_4 + 7^2 c_5 + 9^2 c_6 &= 0, \quad c_1 - c_2 - 3^3 c_3 - 5^3 c_4 - 7^3 c_5 - 9^3 c_6 = 0, \\ c_1 + c_2 + 3^4 c_3 + 5^4 c_4 + 7^4 c_5 + 9^4 c_6 &= 0, \quad c_1 - c_2 - 3^5 c_3 - 5^5 c_4 - 7^5 c_5 - 9^5 c_6 = 0, \end{aligned}$$

we get $c_1 = 63/256$, $c_2 = 315/256$, $c_3 = -210/256$, $c_4 = 126/256$, $c_5 = -45/256$, $c_6 = 7/256$.

From these values, we obtain the following formula :

$$f(0) = (63f(h/2) + 315f(-h/2) - 210f(-3h/2) + 126f(-5h/2) - 45f(-7h/2) + 7f(-9h/2))/256 - 21h^6 f^{(6)}(\xi)/1024. \quad (6.3)$$

7. Seven-node interpolation

(1) 7_{-4}^{+3} formula.

From

$$\begin{aligned} \sum_{k=1}^7 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) - 7c_7 = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2 c_7 &= 0, \quad c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) - 7^3 c_7 = 0, \\ c_1 + c_2 + 3^4(c_3 + c_4) + 5^4(c_5 + c_6) + 7^4 c_7 &= 0, \quad c_1 - c_2 + 3^5(c_3 - c_4) + 5^5(c_5 - c_6) - 7^5 c_7 = 0, \end{aligned}$$

$$c_1 + c_2 + 3^6(c_3 + c_4) + 5^6(c_5 + c_6) + 7^6 c_7 = 0.$$

we get

$$\begin{aligned} c_1 &= 525/1024, \quad c_2 = 700/1024, \quad c_3 = -70/1024, \quad c_4 = -175/1024, \quad c_5 = 7/1024, \\ c_6 &= 42/1024, \quad c_7 = -5/1024. \end{aligned}$$

Using these values, we obtain the following formula :

$$\begin{aligned} f(0) &= (525f(h/2) + 700f(-h/2) - 70f(3h/2) - 175f(-3h/2) + 7f(5h/2) + 42f(-5h/2) \\ &\quad - 5f(-7h/2))/1024 - 5h^7 f^{(7)}(\xi)/2048. \end{aligned} \quad (7.1)$$

(2) 7_{-5}^{+2} formula.

From the following equations :

$$\begin{aligned} \sum_{k=1}^7 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) - 5c_5 - 7c_6 - 9c_7 = 0, \quad c_1 + c_2 + 3^2(c_3 + c_4) + 5^2c_5 + 7^2c_6 + 9^2c_7 = 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) - 5^3c_5 - 7^3c_6 - 9^3c_7 &= 0, \quad c_1 + c_2 + 3^4(c_3 + c_4) + 5^4c_5 + 7^4c_6 + 9^4c_7 = 0, \\ c_1 - c_2 + 3^5(c_3 - c_4) - 5^5c_5 - 7^5c_6 - 9^5c_7 &= 0, \quad c_1 + c_2 + 3^6(c_3 + c_4) + 5^6c_5 + 7^6c_6 + 9^6c_7 = 0, \end{aligned}$$

we get the following values : $c_1 = 378/1024, c_2 = 945/1024, c_3 = -21/1024, c_4 = -420/1024, c_5 = 189/1024, c_6 = -54/1024, c_7 = 7/1024.$

Using these values, we obtain the following formula :

$$\begin{aligned} f(0) &= (378f(h/2) + 945f(-h/2) - 21f(3h/2) - 420f(-3h/2) + 189f(-5h/2) - 54f(-7h/2) \\ &\quad + 7f(-9h/2))/1024 + 9h^7 f^{(7)}(\xi)/2048. \end{aligned} \quad (7.2)$$

(3) 7_{-6}^{+1} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^7 c_k &= 1, \quad c_1 - c_2 - 3c_3 - 5c_4 - 7c_5 - 9c_6 - 11c_7 = 0, \\ c_1 + c_2 + 3^2c_3 + 5^2c_4 + 7^2c_5 + 9^2c_6 + 11^2c_7 &= 0, \quad c_1 - c_2 - 3^3c_3 - 5^3c_4 - 7^3c_5 - 9^3c_6 - 11^3c_7 = 0, \\ c_1 + c_2 + 3^4c_3 + 5^4c_4 + 7^4c_5 + 9^4c_6 + 11^4c_7 &= 0, \quad c_1 - c_2 - 3^5c_3 - 5^5c_4 - 7^5c_5 - 9^5c_6 - 11^5c_7 = 0, \\ c_1 + c_2 + 3^6c_3 + 5^6c_4 + 7^6c_5 + 9^6c_6 + 11^6c_7 &= 0, \end{aligned}$$

we get $c_1 = 231/1024, c_2 = 1386/1024, c_3 = -1155/1024, c_4 = 924/1024, c_5 = -495/1024, c_6 = 154/1024, c_7 = -21/1024.$

From these values, we obtain the following formula :

$$\begin{aligned} f(0) &= (231f(h/2) + 1386f(-h/2) - 1155f(-3h/2) + 924f(-5h/2) - 495f(-7h/2) \\ &\quad + 154f(-9h/2) - 21f(-11h/2))/1024 - 33h^7 f^{(7)}(\xi)/2048. \end{aligned} \quad (7.3)$$

8. Eight-node interpolation

(1) 8_{-4}^{+4} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^4 c_k &= 1/2, \quad c_1 + 3^2c_2 + 5^2c_3 + 7^2c_4 = 0, \\ c_1 + 3^4c_2 + 5^4c_3 + 7^4c_4 &= 0, \quad c_1 + 3^6c_2 + 5^6c_3 + 7^6c_4 = 0, \end{aligned}$$

we have $c_1 = 1225/2048, c_2 = -245/2048, c_3 = 49/2048, c_4 = -5/2048.$

Using these values, we obtain the following formula :

$$f(0) = (1225(f(h/2) + f(-h/2)) - 245(f(3h/2) + f(-3h/2)) + 49(f(5h/2) + f(-5h/2)) - 5(f(7h/2) + f(-7h/2)))/2048 + 35h^8 f^{(8)}(\xi)/65536. \quad (8.1)$$

(2) 8_{-5}^{+3} formula.

From the following equations :

$$\begin{aligned} \sum_{k=1}^8 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) - 7c_7 - 9c_8 = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2c_7 + 9^2c_8 &= 0, \quad c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) - 7^3c_7 - 9^3c_8 = 0, \\ c_1 + c_2 + 3^4(c_3 + c_4) + 5^4(c_5 + c_6) + 7^4c_7 + 9^4c_8 &= 0, \quad c_1 - c_2 + 3^5(c_3 - c_4) + 5^5(c_5 - c_6) - 7^5c_7 - 9^5c_8 = 0, \\ c_1 + c_2 + 3^6(c_3 + c_4) + 5^6(c_5 + c_6) + 7^6c_7 + 9^6c_8 &= 0, \quad c_1 - c_2 + 3^7(c_3 - c_4) + 5^7(c_5 - c_6) - 7^7c_7 - 9^7c_8 = 0, \end{aligned}$$

we have $c_1 = 945/2048$, $c_2 = 1575/2048$, $c_3 = -105/2048$, $c_4 = -525/2048$, $c_5 = 9/2048$, $c_6 = 189/2048$, $c_7 = -45/2048$, $c_8 = 5/2048$.

Using these values, we obtain the following formula :

$$f(0) = (945f(h/2) + 1575f(-h/2) - 105f(3h/2) - 525f(-3h/2) + 9f(5h/2) + 189f(-5h/2) - 45f(-7h/2) + 5f(-9h/2))/2048 - 45h^8 f^{(8)}(\xi)/32768. \quad (8.2)$$

(3) 8_{-6}^{+2} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^8 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) - 5c_5 - 7c_6 - 9c_7 - 11c_8 = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2c_5 + 7^2c_6 + 9^2c_7 + 11^2c_8 &= 0, \quad c_1 - c_2 + 3^3(c_3 - c_4) - 5^3c_5 - 7^3c_6 - 9^3c_7 - 11^3c_8 = 0, \\ c_1 + c_2 + 3^4(c_3 + c_4) + 5^4c_5 + 7^4c_6 + 9^4c_7 + 11^4c_8 &= 0, \quad c_1 - c_2 + 3^5(c_3 - c_4) - 5^5c_5 - 7^5c_6 - 9^5c_7 - 11^5c_8 = 0, \\ c_1 + c_2 + 3^6(c_3 + c_4) + 5^6c_5 + 7^6c_6 + 9^6c_7 + 11^6c_8 &= 0, \quad c_1 - c_2 + 3^7(c_3 - c_4) - 5^7c_5 - 7^7c_6 - 9^7c_7 - 11^7c_8 = 0, \end{aligned}$$

we have $c_1 = 693/2048$, $c_2 = 2079/2048$, $c_3 = -33/2048$, $c_4 = -1155/2048$, $c_5 = 693/2048$, $c_6 = -297/2048$, $c_7 = 77/2048$, $c_8 = -9/2048$.

Using these values, we obtain the following formula :

$$f(0) = (693f(h/2) + 2079f(-h/2) - 33f(3h/2) - 1155f(-3h/2) + 693f(-5h/2) - 297f(-7h/2) + 77f(-9h/2) - 9f(-11h/2))/2048 + 99h^8 f^{(8)}(\xi)/32768. \quad (8.3)$$

(4) 8_{-7}^{+1} formula.

Solving following equations :

$$\begin{aligned} \sum_{k=1}^8 c_k &= 1, \quad c_1 - c_2 - 3c_3 - 5c_4 - 7c_5 - 9c_6 - 11c_7 - 13c_8 = 0, \\ c_1 + c_2 + 3^2c_3 + 5^2c_4 + 7^2c_5 + 9^2c_6 + 11^2c_7 + 13^2c_8 &= 0, \quad c_1 - c_2 - 3^3c_3 - 5^3c_4 - 7^3c_5 - 9^3c_6 - 11^3c_7 - 13^3c_8 = 0, \\ c_1 + c_2 + 3^4c_3 + 5^4c_4 + 7^4c_5 + 9^4c_6 + 11^4c_7 + 13^4c_8 &= 0, \quad c_1 - c_2 - 3^5c_3 - 5^5c_4 - 7^5c_5 - 9^5c_6 - 11^5c_7 - 13^5c_8 = 0, \\ c_1 + c_2 + 3^6c_3 + 5^6c_4 + 7^6c_5 + 9^6c_6 + 11^6c_7 + 13^6c_8 &= 0, \quad c_1 - c_2 - 3^7c_3 - 5^7c_4 - 7^7c_5 - 9^7c_6 - 11^7c_7 - 13^7c_8 = 0, \end{aligned}$$

we obtain $c_1 = 429/2048$, $c_2 = -c_3 = c_4 = 3003/2048$, $c_5 = -2145/2048$, $c_6 = 1001/2048$, $c_7 = -273/2048$, $c_8 = 33/2048$.

From these values, we have the following formula :

$$f(0) = (429f(h/2) + 3003(f(-h/2) - f(-3h/2) + f(-5h/2)) - 2145f(-7h/2) + 1001f(-9h/2) - 273f(-11h/2) + 33f(-13h/2))/2048 + 33h^8 f^{(8)}(\xi)/32768.$$

$$-273f(-11h/2) + 33f(-13h/2))/2048 - 429h^8 f^{(8)}(\xi)/32768. \quad (8.4)$$

9. Nine-node interpolation

(1) 9_{-5}^{+4} formula.

We solve the following equations :

$$\begin{aligned} \sum_{k=1}^9 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) + 7(c_7 - c_8) - 9c_9 = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2(c_7 + c_8) + 9^2c_9 &= 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) + 7^3(c_7 - c_8) - 9^3c_9 &= 0, \\ c_1 + c_2 + 3^4(c_3 + c_4) + 5^4(c_5 + c_6) + 7^4(c_7 + c_8) + 9^4c_9 &= 0, \\ c_1 - c_2 + 3^5(c_3 - c_4) + 5^5(c_5 - c_6) + 7^5(c_7 - c_8) - 9^5c_9 &= 0, \\ c_1 + c_2 + 3^6(c_3 + c_4) + 5^6(c_5 + c_6) + 7^6(c_7 + c_8) + 9^6c_9 &= 0, \\ c_1 - c_2 + 3^7(c_3 - c_4) + 5^7(c_5 - c_6) + 7^7(c_7 - c_8) - 9^7c_9 &= 0, \\ c_1 + c_2 + 3^8(c_3 + c_4) + 5^8(c_5 + c_6) + 7^8(c_7 + c_8) + 9^8c_9 &= 0, \end{aligned}$$

and we get $c_1 = 17640/32768$, $c_2 = 22050/32768$, $c_3 = -2940/32768$, $c_4 = -5880/32768$,
 $c_5 = 504/32768$, $c_6 = 1764/32768$, $c_7 = -45/32768$, $c_8 = -360/32768$, $c_9 = 35/32768$.

Using these values, we obtain the following formula :

$$\begin{aligned} f(0) &= (17640f(h/2) + 22050f(-h/2) - 2940f(3h/2) - 5880f(-3h/2) + 504f(5h/2) + 1764f(-5h/2) \\ &\quad - 45f(7h/2) - 360f(-7h/2) + 35f(-9h/2))/32768 + 35h^9 f^{(9)}(\xi)/65536. \end{aligned} \quad (9.1)$$

(2) 9_{-6}^{+3} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^9 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) - 7c_7 - 9c_8 - 11c_9 = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2c_7 + 9^2c_8 + 11^2c_9 &= 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) - 7^3c_7 - 9^3c_8 - 11^3c_9 &= 0, \\ c_1 + c_2 + 3^4(c_3 + c_4) + 5^4(c_5 + c_6) + 7^4c_7 + 9^4c_8 + 11^4c_9 &= 0, \\ c_1 - c_2 + 3^5(c_3 - c_4) + 5^5(c_5 - c_6) - 7^5c_7 - 9^5c_8 - 11^5c_9 &= 0, \\ c_1 + c_2 + 3^6(c_3 + c_4) + 5^6(c_5 + c_6) + 7^6c_7 + 9^6c_8 + 11^6c_9 &= 0, \\ c_1 - c_2 + 3^7(c_3 - c_4) + 5^7(c_5 - c_6) - 7^7c_7 - 9^7c_8 - 11^7c_9 &= 0, \\ c_1 + c_2 + 3^8(c_3 + c_4) + 5^8(c_5 + c_6) + 7^8c_7 + 9^8c_8 + 11^8c_9 &= 0, \end{aligned}$$

we get $c_1 = 13860/32768$, $c_2 = 27720/32768$, $c_3 = -1320/32768$, $c_4 = -11550/32768$,
 $c_5 = 99/32768$, $c_6 = 5544/32768$, $c_7 = -1980/32768$, $c_8 = 440/32768$, $c_9 = -45/32768$.

From these values, we obtain the following formula :

$$\begin{aligned} f(0) &= (13860f(h/2) + 27720f(-h/2) - 1320f(3h/2) - 11550f(-3h/2) + 99f(5h/2) + 5544f(-5h/2) \\ &\quad - 1980f(-7h/2) + 440f(-9h/2) - 45f(-11h/2))/32768 - 55h^9 f^{(9)}(\xi)/65536. \end{aligned} \quad (9.2)$$

(3) 9_{-7}^{+2} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^9 c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) - 5c_5 - 7c_6 - 9c_7 - 11c_8 - 13c_9 = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2c_5 + 7^2c_6 + 9^2c_7 + 11^2c_8 + 13^2c_9 &= 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) - 5^3c_5 - 7^3c_6 - 9^3c_7 - 11^3c_8 - 13^3c_9 &= 0, \end{aligned}$$

$$\begin{aligned}
c_1 + c_2 + 3^4(c_3 + c_4) + 5^4c_5 + 7^4c_6 + 9^4c_7 + 11^4c_8 + 13^4c_9 &= 0, \\
c_1 - c_2 + 3^5(c_3 - c_4) - 5^5c_5 - 7^5c_6 - 9^5c_7 - 11^5c_8 - 13^5c_9 &= 0, \\
c_1 + c_2 + 3^6(c_3 + c_4) + 5^6c_5 + 7^6c_6 + 9^6c_7 + 11^6c_8 + 13^6c_9 &= 0, \\
c_1 - c_2 + 3^7(c_3 - c_4) - 5^7c_5 - 7^7c_6 - 9^7c_7 - 11^7c_8 - 13^7c_9 &= 0, \\
c_1 + c_2 + 3^8(c_3 + c_4) + 5^8c_5 + 7^8c_6 + 9^8c_7 + 11^8c_8 + 13^8c_9 &= 0,
\end{aligned}$$

we have $c_1 = 10296/32768$, $c_2 = 36036/32768$, $c_3 = -429/32768$, $c_4 = -24024/32768$,
 $c_5 = 18018/32768$, $c_6 = -10296/32768$, $c_7 = 4004/32768$, $c_8 = -936/32768$, $c_9 = 99/32768$.

Using theses values, we obtain the following formula :

$$\begin{aligned}
f(0) &= (10296f(h/2) + 36036f(-h/2) - 429f(3h/2) - 24024f(-3h/2) + 18018f(-5h/2) \\
&\quad - 10296f(-7h/2) + 4004f(-9h/2) - 936f(-11h/2) + 99f(-9h/2))/32768 \\
&\quad + 143h^9 f^{(9)}(\xi)/65536.
\end{aligned} \tag{9.3}$$

(4) 9_{-8}^{+1} formula.

Solving the following equations :

$$\begin{aligned}
\sum_{k=1}^9 c_k &= 1, \quad c_1 - c_2 - 3c_3 - 5c_4 - 7c_5 - 9c_6 - 11c_7 - 13c_8 - 15c_9 = 0, \\
c_1 + c_2 + 3^2c_3 + 5^2c_4 + 7^2c_5 + 9^2c_6 + 11^2c_7 + 13^2c_8 + 15^2c_9 &= 0, \\
c_1 - c_2 - 3^3c_3 - 5^3c_4 - 7^3c_5 - 9^3c_6 - 11^3c_7 - 13^3c_8 - 15^3c_9 &= 0, \\
c_1 + c_2 + 3^4c_3 + 5^4c_4 + 7^4c_5 + 9^4c_6 + 11^4c_7 + 13^4c_8 + 15^4c_9 &= 0, \\
c_1 - c_2 - 3^5c_3 - 5^5c_4 - 7^5c_5 - 9^5c_6 - 11^5c_7 - 13^5c_8 - 15^5c_9 &= 0, \\
c_1 + c_2 + 3^6c_3 + 5^6c_4 + 7^6c_5 + 9^6c_6 + 11^6c_7 + 13^6c_8 + 15^6c_9 &= 0, \\
c_1 - c_2 - 3^7c_3 - 5^7c_4 - 7^7c_5 - 9^7c_6 - 11^7c_7 - 13^7c_8 - 15^7c_9 &= 0, \\
c_1 + c_2 + 3^8c_3 + 5^8c_4 + 7^8c_5 + 9^8c_6 + 11^8c_7 + 13^8c_8 + 15^8c_9 &= 0,
\end{aligned}$$

we have $c_1 = 6435/32768$, $c_2 = 51480/32768$, $c_3 = -60060/32768$, $c_4 = 72072/32768$,
 $c_5 = -64350/32768$, $c_6 = 40040/32768$, $c_7 = -16380/32768$, $c_8 = 3960/32768$, $c_9 = -429/32768$.

Using these values, we obtain the following formula :

$$\begin{aligned}
f(0) &= (6435f(h/2) + 51480f(-h/2) - 60060f(-3h/2) + 72072f(-5h/2) - 64350f(-7h/2) \\
&\quad + 40040f(-9h/2) - 16380f(-11h/2) + 3960f(-13h/2) - 429f(-15h/2))/32768 \\
&\quad - 715h^9 f^{(9)}(\xi)/65536.
\end{aligned} \tag{9.4}$$

10. Eleven-node interpolation

(1) 11_{-6}^{+5} formula.

Solving the following equations :

$$\begin{aligned}
\sum_{k=1}^{11} c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) + 7(c_7 - c_8) + 9(c_9 - c_{10}) - 11c_{11} = 0, \\
c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2(c_7 + c_8) + 9^2(c_9 + c_{10}) + 11^2c_{11} &= 0, \\
c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) + 7^3(c_7 - c_8) + 9^3(c_9 - c_{10}) - 11^3c_{11} &= 0, \\
c_1 + c_2 + 3^4(c_3 + c_4) + 5^4(c_5 + c_6) + 7^4(c_7 + c_8) + 9^4(c_9 + c_{10}) + 11^4c_{11} &= 0, \\
c_1 - c_2 + 3^5(c_3 - c_4) + 5^5(c_5 - c_6) + 7^5(c_7 - c_8) + 9^5(c_9 - c_{10}) - 11^5c_{11} &= 0, \\
&\dots, \\
c_1 + c_2 + 3^{10}(c_3 + c_4) + 5^{10}(c_5 + c_6) + 7^{10}(c_7 + c_8) + 9^{10}(c_9 + c_{10}) + 11^{10}c_{11} &= 0,
\end{aligned}$$

we have $c_1 = 145530/262144$, $c_2 = 174636/262144$, $c_3 = -27720/262144$, $c_4 = -48510/262144$,

$c_5 = 6237/262144$, $c_6 = 16632/262144$, $c_7 = -990/262144$, $c_8 = -4455/262144$, $c_9 = 77/262144$,
 $c_{10} = 770/262144$, $c_{11} = -63/262144$.

From these values, we get the following formula :

$$\begin{aligned} f(0) = & (145530f(h/2) + 174636f(-h/2) - 27720f(3h/2) - 48510f(-3h/2) + 6237f(5h/2) \\ & + 16632f(-5h/2) - 990f(7h/2) - 4455f(-7h/2) + 77f(9h/2) + 770f(-9h/2) \\ & - 63f(-11h/2))/262144 - 63h^{11}f^{(11)}(\xi)/524288. \end{aligned} \quad (10.1)$$

(2) 11_{-7}^{+4} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^{11} c_k = 1, \quad & c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) + 7(c_7 - c_8) - 9c_9 - 11c_{10} - 13c_{11} = 0, \\ & c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2(c_7 + c_8) + 9^2c_9 + 11^2c_{10} + 13^2c_{11} = 0, \\ & c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) + 7^3(c_7 - c_8) - 9^3c_9 - 11^3c_{10} - 13^3c_{11} = 0, \\ & \dots, \\ & c_1 + c_2 + 3^{10}(c_3 + c_4) + 5^{10}(c_5 + c_6) + 7^{10}(c_7 + c_8) + 9^{10}c_9 + 11^{10}c_{10} + 13^{10}c_{11} = 0, \end{aligned}$$

we obtain $c_1 = 120120/262144$, $c_2 = 210210/262144$, $c_3 = -15015/262144$, $c_4 = -84084/262144$,
 $c_5 = 2002/262144$, $c_6 = 42042/262144$, $c_7 = -143/262144$, $c_8 = -17160/262144$, $c_9 = 5005/262144$,
 $c_{10} = -910/262144$, $c_{11} = 77/262144$.

Using these values, we get the following formula :

$$\begin{aligned} f(0) = & (120120f(h/2) + 210210f(-h/2) - 15015f(3h/2) - 84084f(-3h/2) + 2002f(5h/2) \\ & + 42042f(-5h/2) - 143f(7h/2) - 17160f(-7h/2) + 5005f(-9h/2) - 910f(-11h/2) \\ & + 77f(-13h/2))/262144 + 91h^{11}f^{(11)}(\xi)/524288. \end{aligned} \quad (10.2)$$

(3) 11_{-8}^{+3} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^{11} c_k = 1, \quad & c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) - 7c_7 - 9c_8 - 11c_9 - 13c_{10} - 15c_{11} = 0, \\ & c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2c_7 + 9^2c_8 + 11^2c_9 + 13^2c_{10} + 15^2c_{11} = 0, \\ & c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) - 7^3c_7 - 9^3c_8 - 11^3c_9 - 13^3c_{10} - 15^3c_{11} = 0, \\ & \dots, \\ & c_1 + c_2 + 3^{10}(c_3 + c_4) + 5^{10}(c_5 + c_6) + 7^{10}c_7 + 9^{10}c_8 + 11^{10}c_9 + 13^{10}c_{10} + 15^{10}c_{11} = 0, \end{aligned}$$

we have $c_1 = 96525/262144$, $c_2 = 257400/262144$, $c_3 = -7150/262144$, $c_4 = -150150/262144$,
 $c_5 = 429/262144$, $c_6 = 108108/262144$, $c_7 = -64350/262144$, $c_8 = 28600/262144$, $c_9 = -8775/262144$,
 $c_{10} = 1650/262144$, $c_{11} = -143/262144$.

From these values, we get the following formula :

$$\begin{aligned} f(0) = & (96525f(h/2) + 257400f(-h/2) - 7150f(3h/2) - 150150f(-3h/2) + 429f(5h/2) \\ & + 108108f(-5h/2) - 64350f(-7h/2) + 28600f(-9h/2) - 8775f(-11h/2) + 1650f(-13h/2) \\ & - 143f(-15h/2))/262144 - 195h^{11}f^{(11)}(\xi)/524288. \end{aligned} \quad (10.3)$$

(4) 11_{-9}^{+2} formula.

Solving the following equations :

$$\sum_{k=1}^{11} c_k = 1, \quad c_1 - c_2 + 3(c_3 - c_4) - 5c_5 - 7c_6 - 9c_7 - 11c_8 - 13c_9 - 15c_{10} - 17c_{11} = 0,$$

$$\begin{aligned}
& c_1 + c_2 + 3^2(c_3 + c_4) + 5^2c_5 + 7^2c_6 + 9^2c_7 + 11^2c_8 + 13^2c_9 + 15^2c_{10} + 17^2c_{11} = 0, \\
& \quad \dots, \\
& c_1 + c_2 + 3^{10}(c_3 + c_4) + 5^{10}c_5 + 7^{10}c_6 + 9^{10}c_7 + 11^{10}c_8 + 13^{10}c_9 + 15^{10}c_{10} + 17^{10}c_{11} = 0,
\end{aligned}$$

we have $c_1 = 72930/262144$, $c_2 = 328185/262144$, $c_3 = -2431/262144$, $c_4 = -291720/262144$,
 $c_5 = 306306/262144$, $c_6 = -262548/262144$, $c_7 = 170170/262144$, $c_8 = -79560/262144$,
 $c_9 = 25245/262144$, $c_{10} = -4862/262144$, $c_{11} = 429/262144$.

From these values, we obtain the following formula :

$$\begin{aligned}
f(0) = & (72930f(h/2) + 328185f(-h/2) - 2431f(3h/2) - 291720f(-3h/2) + 306306f(-5h/2) \\
& - 262548f(-7h/2) + 170170f(-9h/2) - 79560f(-11h/2) + 25245f(-13h/2) - 4862f(-15h/2) \\
& + 429f(-17h/2))/262144 + 663h^{11}f^{(11)}(\xi)/524288.
\end{aligned} \tag{10.4}$$

(5) 11_{-10}^{+1} formula.

From the following equations :

$$\begin{aligned}
& \sum_{k=1}^{11} c_k = 1, \quad c_1 - c_2 - 3c_3 - 5c_4 - 7c_5 - 9c_6 - 11c_7 - 13c_8 - 15c_9 - 17c_{10} - 19c_{11} = 0, \\
& \quad c_1 + c_2 + 3^2c_3 + 5^2c_4 + 7^2c_5 + 9^2c_6 + 11^2c_7 + 13^2c_8 + 15^2c_9 + 17^2c_{10} + 19^2c_{11} = 0, \\
& \quad c_1 - c_2 - 3^3c_3 - 5^3c_4 - 7^3c_5 - 9^3c_6 - 11^3c_7 - 13^3c_8 - 15^3c_9 - 17^3c_{10} - 19^3c_{11} = 0, \\
& \quad \dots, \\
& c_1 + c_2 + 3^{10}c_3 + 5^{10}c_4 + 7^{10}c_5 + 9^{10}c_6 + 11^{10}c_7 + 13^{10}c_8 + 15^{10}c_9 + 17^{10}c_{10} + 19^{10}c_{11} = 0,
\end{aligned}$$

we have $c_1 = 46189/262144$, $c_2 = 461890/262144$, $c_3 = -692835/262144$, $c_4 = 1108536/262144$,
 $c_5 = -1385670/262144$, $c_6 = 1293292/262144$, $c_7 = -881790/262144$, $c_8 = 426360/262144$,
 $c_9 = -138567/262144$, $c_{10} = 27170/262144$, $c_{11} = -2431/262144$.

Using these values, we obtain the following formula :

$$\begin{aligned}
f(0) = & (46189f(h/2) + 461890f(-h/2) - 692835f(-3h/2) + 1108536f(-5h/2) - 1385670f(-7h/2) \\
& + 1293292f(-9h/2) - 881790f(-11h/2) + 426360f(-13h/2) - 138567f(-15h/2) \\
& + 27170f(-17h/2) - 2431f(-19h/2))/262144 - 4199h^{11}f^{(11)}(\xi)/524288.
\end{aligned} \tag{10.5}$$

11. Thirteen-node interpolation

(1) 13_{-7}^{+6} formula.

Solving following simultaneous equations :

$$\begin{aligned}
& \sum_{k=1}^{13} c_k = 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) + 7(c_7 - c_8) + 9(c_9 - c_{10}) + 11(c_{11} - c_{12}) - 13c_{13} = 0, \\
& \quad c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2(c_7 + c_8) + 9^2(c_9 + c_{10}) + 11^2(c_{11} + c_{12}) + 13^2c_{13} = 0, \\
& \quad c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) + 7^3(c_7 - c_8) + 9^3(c_9 - c_{10}) + 11^3(c_{11} - c_{12}) - 13^3c_{13} = 0, \\
& \quad \dots, \\
& c_1 + c_2 + 3^{12}(c_3 + c_4) + 5^{12}(c_5 + c_6) + 7^{12}(c_7 + c_8) + 9^{12}(c_9 + c_{10}) + 11^{12}(c_{11} + c_{12}) + 13^{12}c_{13} = 0,
\end{aligned}$$

we have $c_0 = 4194304$, $c_1 = 2378376/c_0$, $c_2 = 2774772/c_0$, $c_3 = -495495/c_0$, $c_4 = -792792/c_0$,
 $c_5 = 132132/c_0$, $c_6 = 297297/c_0$, $c_7 = -28314/c_0$, $c_8 = -94380/c_0$, $c_9 = 4004/c_0$, $c_{10} = 22022/c_0$,
 $c_{11} = -273/c_0$, $c_{12} = -3276/c_0$, $c_{13} = 231/c_0$.

From these values, we obtain the following 13_{-7}^{+6} formula :

$$f(0) = (2378376f(h/2) + 2774772f(-h/2) - 495495f(3h/2) - 792792f(-3h/2) + 132132f(5h/2)$$

$$+ 297297f(-5h/2) - 28314f(7h/2) - 94380f(-7h/2) + 4004f(9h/2) + 22022f(-9h/2) \\ - 273f(11h/2) - 3276f(-11h/2) + 231f(-13h/2))/4194304 + 231h^{13}f^{(13)}(\xi)/8388608. \quad (11.1)$$

(2) 13_{-8}^{+5} formula.

From

$$\begin{aligned} \sum_{k=1}^{13} c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) + 7(c_7 - c_8) + 9(c_9 - c_{10}) - 11c_{11} - 13c_{12} - 15c_{13} = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2(c_7 + c_8) + 9^2(c_9 + c_{10}) + 11^2c_{11} + 13^2c_{12} + 15^2c_{13} &= 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) + 7^3(c_7 - c_8) + 9^3(c_9 - c_{10}) - 11^3c_{11} - 13^3c_{12} - 15^3c_{13} &= 0, \\ &\dots, \\ c_1 + c_2 + 3^{12}(c_3 + c_4) + 5^{12}(c_5 + c_6) + 7^{12}(c_7 + c_8) + 9^{12}(c_9 + c_{10}) + 11^{12}c_{11} + 13^{12}c_{12} + 15^{12}c_{13} &= 0, \end{aligned}$$

we have $c_0 = 4194304$, $c_1 = 2027025/c_0$, $c_2 = 3243240/c_0$, $c_3 = -300300/c_0$, $c_4 = -1261260/c_0$, $c_5 = 54054/c_0$, $c_6 = 648648/c_0$, $c_7 = -7020/c_0$, $c_8 = -289575/c_0$, $c_9 = 455/c_0$, $c_{10} = 100100/c_0$, $c_{11} = -24570/c_0$, $c_{12} = 3780/c_0$, $c_{13} = -273/c_0$.

Using these values, we obtain the following interpolation formula :

$$\begin{aligned} f(0) &= (2027025f(h/2) + 3243240f(-h/2) - 300300f(3h/2) - 1261260f(-3h/2) + 54054f(5h/2) \\ &+ 648648f(-5h/2) - 7020f(7h/2) - 289575f(-7h/2) + 455f(9h/2) + 100100f(-9h/2) \\ &- 24570f(-11h/2) + 3780f(-13h/2) - 273f(-15h/2))/4194304 - 315h^{13}f^{(13)}(\xi)/8388608. \quad (11.2) \end{aligned}$$

(3) 13_{-9}^{+4} formula.

From

$$\begin{aligned} \sum_{k=1}^{13} c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) + 7(c_7 - c_8) - 9c_9 - 11c_{10} - 13c_{11} - 15c_{12} - 17c_{13} = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2(c_7 + c_8) + 9^2c_9 + 11^2c_{10} + 13^2c_{11} + 15^2c_{12} + 17^2c_{13} &= 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) + 7^3(c_7 - c_8) - 9^3c_9 - 11^3c_{10} - 13^3c_{11} - 15^3c_{12} - 17^3c_{13} &= 0, \\ &\dots, \\ c_1 + c_2 + 3^{12}(c_3 + c_4) + 5^{12}(c_5 + c_6) + 7^{12}(c_7 + c_8) + 9^{12}c_9 + 11^{12}c_{10} + 13^{12}c_{11} + 15^{12}c_{12} + 17^{12}c_{13} &= 0, \end{aligned}$$

we get $c_0 = 4194304$, $c_1 = 1701700/c_0$, $c_2 = 3828825/c_0$, $c_3 = -170170/c_0$, $c_4 = -2042040/c_0$, $c_5 = 18564/c_0$, $c_6 = 1429428/c_0$, $c_7 = -1105/c_0$, $c_8 = -875160/c_0$, $c_9 = 425425/c_0$, $c_{10} = -154700/c_0$, $c_{11} = 39270/c_0$, $c_{12} = -6188/c_0$, $c_{13} = 455/c_0$.

Using these values, we obtain the following interpolation formula :

$$\begin{aligned} f(0) &= (1701700f(h/2) + 3828825f(-h/2) - 170170f(3h/2) - 2042040f(-3h/2) + 18564f(5h/2) \\ &+ 1429428f(-5h/2) - 1105f(7h/2) - 875160f(-7h/2) + 425425f(-9h/2) - 154700f(-11h/2) \\ &+ 39270f(-13h/2) - 6188f(-15h/2) + 455f(-17h/2))/4194304 + 595h^{13}f^{(13)}(\xi)/8388608. \quad (11.3) \end{aligned}$$

(4) 13_{-10}^{+3} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^{13} c_k &= 1, \quad c_1 - c_2 + 3(c_3 - c_4) + 5(c_5 - c_6) - 7c_7 - 9c_8 - 11c_9 - 13c_{10} - 15c_{11} - 17c_{12} - 19c_{13} = 0, \\ c_1 + c_2 + 3^2(c_3 + c_4) + 5^2(c_5 + c_6) + 7^2c_7 + 9^2c_8 + 11^2c_9 + 13^2c_{10} + 15^2c_{11} + 17^2c_{12} + 19^2c_{13} &= 0, \\ c_1 - c_2 + 3^3(c_3 - c_4) + 5^3(c_5 - c_6) - 7^3c_7 - 9^3c_8 - 11^3c_9 - 13^3c_{10} - 15^3c_{11} - 17^3c_{12} - 19^3c_{13} &= 0, \\ &\dots, \\ c_1 + c_2 + 3^{12}(c_3 + c_4) + 5^{12}(c_5 + c_6) + 7^{12}c_7 + 9^{12}c_8 + 11^{12}c_9 + 13^{12}c_{10} + 15^{12}c_{11} + 17^{12}c_{12} + 19^{12}c_{13} &= 0, \end{aligned}$$

we get $c_0 = 4194304$, $c_1 = 1385670/c_0$, $c_2 = 4618900/c_0$, $c_3 = -83980/c_0$, $c_4 = -3464175/c_0$,
 $c_5 = 4199/c_0$, $c_6 = 3325608/c_0$, $c_7 = -2771340/c_0$, $c_8 = 1847560/c_0$, $c_9 = -944775/c_0$,
 $c_{10} = 355300/c_0$, $c_{11} = -92378/c_0$, $c_{12} = 14820/c_0$, $c_{13} = -1105/c_0$.

Using these values, we obtain the following interpolation formula :

$$\begin{aligned} f(0) = & (1385670f(h/2) + 4618900f(-h/2) - 83980f(3h/2) - 3464175f(-3h/2) + 4199f(5h/2) \\ & + 3325608f(-5h/2) - 2771340f(-7h/2) + 1847560f(-9h/2) - 944775f(-11h/2) \\ & + 355300f(-13h/2) - 92378f(-15h/2) + 14820f(-17h/2) - 1105f(-19h/2))/4194304 \\ & - 1615h^{13}f^{(13)}(\xi)/8388608. \end{aligned} \quad (11.4)$$

(5) 13_{-11}^{+2} formula.

Solving the following equations :

$$\begin{aligned} \sum_{k=1}^{13} c_k = 1, \quad & c_1 - c_2 + 3(c_3 - c_4) - 5c_5 - 7c_6 - 9c_7 - 11c_8 - 13c_9 - 15c_{10} - 17c_{11} - 19c_{12} - 21c_{13} = 0, \\ & c_1 + c_2 + 3^2(c_3 + c_4) + 5^2c_5 + 7^2c_6 + 9^2c_7 + 11^2c_8 + 13^2c_9 + 15^2c_{10} + 17^2c_{11} + 19^2c_{12} + 21^2c_{13} = 0, \\ & c_1 - c_2 + 3^3(c_3 - c_4) - 5^3c_5 - 7^3c_6 - 9^3c_7 - 11^3c_8 - 13^3c_9 - 15^3c_{10} - 17^3c_{11} - 19^3c_{12} - 21^3c_{13} = 0, \\ & \dots, \\ & c_1 + c_2 + 3^{12}(c_3 + c_4) + 5^{12}c_5 + 7^{12}c_6 + 9^{12}c_7 + 11^{12}c_8 + 13^{12}c_9 + 15^{12}c_{10} + 17^{12}c_{11} + 19^{12}c_{12} + 21^{12}c_{13} = 0, \end{aligned}$$

we have $c_0 = 4194304$, $c_1 = 1058148/c_0$, $c_2 = 5819814/c_0$, $c_3 = -29393/c_0$, $c_4 = -6466460/c_0$,
 $c_5 = 8729721/c_0$, $c_6 = -9976824/c_0$, $c_7 = 9053044/c_0$, $c_8 = -6348888/c_0$, $c_9 = 3357585/c_0$,
 $c_{10} = -1293292/c_0$, $c_{11} = 342342/c_0$, $c_{12} = -55692/c_0$, $c_{13} = 4199/c_0$.

Using these values, we obtain the following interpolation formula :

$$\begin{aligned} f(0) = & (1058148f(h/2) + 5819814f(-h/2) - 29393f(3h/2) - 6466460f(-3h/2) + 8729721f(-5h/2) \\ & + 9976824f(-7h/2) + 9053044f(-9h/2) - 6348888f(-11h/2) + 3357585f(-13h/2) \\ & - 1293292f(-15h/2) + 342342f(-17h/2) - 55692f(-19h/2) + 4199f(-21h/2))/4194304 \\ & + 6783h^{13}f^{(13)}(\xi)/8388608. \end{aligned} \quad (11.5)$$

(6) 13_{-12}^{+1} formula.

From

$$\begin{aligned} \sum_{k=1}^{13} c_k = 1, \quad & c_1 - c_2 - 3c_3 - 5c_4 - 7c_5 - 9c_6 - 11c_7 - 13c_8 - 15c_9 - 17c_{10} - 19c_{11} - 21c_{12} - 23c_{13} = 0, \\ & c_1 + c_2 + 3^2c_3 + 5^2c_4 + 7^2c_5 + 9^2c_6 + 11^2c_7 + 13^2c_8 + 15^2c_9 + 17^2c_{10} + 19^2c_{11} + 21^2c_{12} + 23^2c_{13} = 0, \\ & c_1 - c_2 - 3^3c_3 - 5^3c_4 - 7^3c_5 - 9^3c_6 - 11^3c_7 - 13^3c_8 - 15^3c_9 - 17^3c_{10} - 19^3c_{11} - 21^3c_{12} - 23^3c_{13} = 0, \\ & \dots, \\ & c_1 + c_2 + 3^{12}c_3 + 5^{12}c_4 + 7^{12}c_5 + 9^{12}c_6 + 11^{12}c_7 + 13^{12}c_8 + 15^{12}c_9 + 17^{12}c_{10} + 19^{12}c_{11} + 21^{12}c_{12} + 23^{12}c_{13} = 0, \end{aligned}$$

we have $c_0 = 4194304$, $c_1 = 676039/c_0$, $c_2 = 8112468/c_0$, $c_3 = -14872858/c_0$, $c_4 = 29745716/c_0$,
 $c_5 = -47805615/c_0$, $c_6 = 59491432/c_0$, $c_7 = -56787276/c_0$, $c_8 = 41186376/c_0$, $c_9 = -22309287/c_0$,
 $c_{10} = 8748740/c_0$, $c_{11} = -2348346/c_0$, $c_{12} = 386308/c_0$, $c_{13} = -29393/c_0$.

Using these values, we obtain the following interpolation formula :

$$\begin{aligned} f(0) = & (676039f(h/2) + 8112468f(-h/2) - 14872858f(-3h/2) + 29745716f(-5h/2) \\ & - 47805615f(-7h/2) + 59491432f(-9h/2) - 56787276f(-11h/2) + 41186376f(-13h/2) \\ & - 22309287f(-15h/2) + 8748740f(-17h/2) - 2348346f(-19h/2) + 386308f(-21h/2) \\ & - 29393f(-23h/2))/4194304 - 52003h^{13}f^{(13)}(\xi)/8388608. \end{aligned} \quad (11.6)$$

12. Numerical example

From Table 1 to Table 6, we solve numerically the differential equation $y' = x^3 y$, with the initial value $y(0) = 1$, the step size $h = 0.0625$, and the analytical solution is $y = \exp(x^4/4)$, y_t is the value of the analytical solution, and $E_y = (y - y_t) \times 10^{10}$ in Table 1 and in Table 2, and $E_y = (y - y_t) \times 10^{12}$ in other Tables.

In Table 1 and Table 2, we use five-node predictor and correctors. In Table 3 and Table 4, we use seven-node formulas. And, in Table 5 and Table 6, we use nine-node formulas.

In Table 1, Table 3, and Table 5, if we can't correct y_n after at most 4 times correction, we halve the step size h . In Table 2, Table 4, and Table 6, if we can't correct y_n after at most 5 times correction, we halve the step size h .

When we halve the step size h , we show the half value between x_n and x_{n+1} by $x_{n.5}$, and the half value between y_n and y_{n+1} by $y_{n.5}$. When we halve the step size h we show the first approximate values of $y_{n.5}$ for $x_{n.5}$ using interpolation formulas.

13. Conclusion

In Tables 1, 2, 3 and 4, we got the first approximate values of $y_{n.5}$ using two and more nodes formulas. But, in Table 5 and Table 6, we obtain the first approximate values of $y_{n.5}$ using five and more nodes interpolation formulas.

This reason is due to the fact that the first approximate values of $y_{n.5}$ are different from the true values, so we must correct $y_{n.5}$ many times by correctors, at Table 5 and Table 6. But, in Tables 1, 2, 3 and 4, we have good approximate values for $y_{n.5}$ by 2_{-1}^{+1} .

References

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- [2] F. Tamari, R. Tsukamoto, R. Furuki, and H. Yanagiwara, On a New Multistep Method II, Bull. of Fukuoka Univ. of Ed. part III 49 (2000) 1-6.
- [3] F. Tamari and H. Yanagiwara, On the Ten-Node Interpolation Using Taylor Expansions, Bull. of Fukuoka Univ. of Ed. part III 57 (2008) 41-51.
- [4] F. Tamari and H. Yanagiwara, On the Twelve-Node Interpolation Using Taylor Expansions, Bull. of Fukuoka Univ. of Ed. part III 57 (2008) 53-62.

Table 1 ($y' = x^3y$, five nodes, 4 times) 1/5

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{10}$ | | |
|-----------------|---------|---|-----|---------------|
| S | x | y | E | y' |
| 1 | -0.125 | 1.0000610370 | 0 | -0.0019532442 |
| 2 | -0.0625 | 1.0000038147 | 0 | -0.0002441416 |
| 3 | 0 | 1.0000000000 | 0 | 0.0000000000 |
| 4 | 0.0625 | 1.0000038147 | 0 | 0.0002441416 |
| 5 | 0.125 | 1.0000610370 | 0 | 0.0019532442 |
| 6 | 0.1875 | 1.0003090407 | 25 | 0.0065938340 |
| 7 | 0.25 | 1.0009770445 | 50 | 0.0156402663 |
| 8 | 0.3125 | 1.0023870378 | 76 | 0.0305904247 |

the first approximate value of $y_{6.5}$ for $x_{6.5} = 0.21875$

1.0006430426 by 2_{-1}^{+1} 1.0005502939 by 3_{-1}^{+2}
 1.0005704182 by 4_{-2}^{+2} 1.0005725926 by 5_{-3}^{+2}
 1.0005726034 by 6_{-4}^{+2} 1.0005726080 by 7_{-5}^{+2}
 1.0005726101 by 8_{-6}^{+2} 1.0005726111 by 9_{-7}^{+2}

the first approximate value of $y_{7.5}$ for $x_{7.5} = 0.28125$

1.0016820411 by 2_{-1}^{+1} 1.0015892924 by 3_{-2}^{+1}
 1.0015691681 by 4_{-3}^{+1} 1.0015655442 by 5_{-4}^{+1}
 1.0015655189 by 6_{-5}^{+1} 1.0015655051 by 7_{-6}^{+1}
 1.0015654975 by 8_{-7}^{+1} 1.0015654932 by 9_{-8}^{+1}

| | | | | |
|----|---------|--------------|----|--------------|
| 9 | 0.34375 | 1.0034967908 | 48 | 0.0407609323 |
| 10 | 0.375 | 1.0049560937 | 51 | 0.0529957315 |
| 11 | 0.40625 | 1.0068327151 | 48 | 0.0675052330 |
| 12 | 0.4375 | 1.0092011687 | 77 | 0.0845107424 |
| 13 | 0.46875 | 1.0121430813 | 50 | 0.1042475250 |
| 14 | 0.5 | 1.0157477139 | 53 | 0.1269684642 |

the first approximate value of $y_{12.5}$ for $x_{12.5} = 0.453125$

1.0106721250 by 2_{-1}^{+1} 1.0105892850 by 3_{-1}^{+2}
 1.0105948638 by 4_{-2}^{+2} 1.0105950427 by 5_{-3}^{+2}
 1.0105950488 by 6_{-4}^{+2} 1.0105950499 by 7_{-5}^{+2}
 1.0105950503 by 8_{-6}^{+2} 1.0105950509 by 9_{-7}^{+2}

the first approximate value of $y_{13.5}$ for $x_{13.5} = 0.484375$

1.0139453976 by 2_{-1}^{+1} 1.0138625576 by 3_{-2}^{+1}
 1.0138569788 by 4_{-3}^{+1} 1.0138566806 by 5_{-4}^{+1}
 1.0138566663 by 6_{-5}^{+1} 1.0138566631 by 7_{-6}^{+1}
 1.0138566614 by 8_{-7}^{+1} 1.0138566587 by 9_{-8}^{+1}

| S | x | y | E | y' |
|-----|----------|--------------|-----|--------------|
| 15 | 0.515625 | 1.0178286716 | 50 | 0.1395328864 |
| 16 | 0.53125 | 1.0201125487 | 51 | 0.1529483933 |
| 17 | 0.546875 | 1.0226129424 | 51 | 0.1672536083 |
| 18 | 0.5625 | 1.0253440697 | 54 | 0.1824892155 |
| 19 | 0.578125 | 1.0283207995 | 51 | 0.1986981715 |
| 20 | 0.59375 | 1.0315586931 | 51 | 0.2159259362 |
| 21 | 0.609375 | 1.0350740419 | 51 | 0.2342207225 |
| 22 | 0.625 | 1.0388839147 | 55 | 0.2536337682 |
| 23 | 0.640625 | 1.0430062049 | 52 | 0.2742196299 |
| 24 | 0.65625 | 1.0474596886 | 52 | 0.2960365044 |
| 25 | 0.671875 | 1.0522640803 | 52 | 0.3191465768 |
| 26 | 0.6875 | 1.0574401007 | 56 | 0.3436163999 |
| 27 | 0.703125 | 1.0630095455 | 53 | 0.3695173066 |
| 28 | 0.71875 | 1.0689953675 | 53 | 0.3969258617 |

Table 1 ($y' = x^3y$, five nodes, 4 times) 2/5

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{10}$ | | |
|-----------------|----------|---|-----|--------------|
| S | x | y | E | y' |
| 29 | 0.734375 | 1.0754217585 | 53 | 0.4259243516 |
| 30 | 0.75 | 1.0823142447 | 57 | 0.4566013220 |
| 31 | 0.765625 | 1.0896997874 | 54 | 0.4890521633 |
| 32 | 0.78125 | 1.0976068978 | 55 | 0.5233797540 |
| 33 | 0.796875 | 1.1060657564 | 55 | 0.5596951624 |
| 34 | 0.8125 | 1.1151083487 | 59 | 0.5981184185 |
| 35 | 0.828125 | 1.1247686097 | 56 | 0.6387793591 |

the first approximate value of $y_{33.5}$ for $x_{33.5} = 0.8046875$

1.1105870525 by 2_{-1}^{+1} 1.1105098439 by 3_{-1}^{+2}
 1.1105119649 by 4_{-2}^{+2} 1.1105120106 by 5_{-3}^{+2}
 1.1105120123 by 6_{-4}^{+2} 1.1105120123 by 7_{-5}^{+2}
 1.1105120123 by 8_{-6}^{+2} 1.1105120122 by 9_{-7}^{+2}

the first approximate value of $y_{34.5}$ for $x_{34.5} = 0.8203125$

1.1199384792 by 2_{-1}^{+1} 1.1198612706 by 3_{-2}^{+1}
 1.1198591496 by 4_{-3}^{+1} 1.1198590735 by 5_{-4}^{+1}
 1.1198590695 by 6_{-5}^{+1} 1.1198590694 by 7_{-6}^{+1}
 1.1198590695 by 8_{-7}^{+1} 1.1198590698 by 9_{-8}^{+1}

| | | | | |
|----|-----------|--------------|----|--------------|
| 36 | 0.8359375 | 1.1298415445 | 59 | 0.6599924446 |
| 37 | 0.84375 | 1.1350825884 | 60 | 0.6818185604 |
| 38 | 0.8515625 | 1.1404966064 | 60 | 0.7042771243 |
| 39 | 0.859375 | 1.1460886177 | 57 | 0.7273883582 |
| 40 | 0.8671875 | 1.1518638039 | 61 | 0.7511733275 |
| 41 | 0.875 | 1.1578275129 | 61 | 0.7756539783 |
| 42 | 0.8828125 | 1.1639852685 | 61 | 0.8008531818 |
| 43 | 0.890625 | 1.1703427767 | 59 | 0.8267947763 |
| 44 | 0.8984375 | 1.1769059348 | 62 | 0.8535036152 |
| 45 | 0.90625 | 1.1836808364 | 62 | 0.8810056128 |
| 46 | 0.9140625 | 1.1906737833 | 63 | 0.9093277979 |
| 47 | 0.921875 | 1.1978912922 | 60 | 0.9384983662 |
| 48 | 0.9296875 | 1.2053401061 | 63 | 0.9685467376 |
| 49 | 0.9375 | 1.2130272011 | 64 | 0.9995036142 |
| 50 | 0.9453125 | 1.2209597993 | 64 | 1.0314010444 |
| 51 | 0.953125 | 1.2291453784 | 62 | 1.0642724882 |
| 52 | 0.9609375 | 1.2375916843 | 65 | 1.0981528877 |
| 53 | 0.96875 | 1.2463067400 | 66 | 1.1330787382 |
| 54 | 0.9765625 | 1.2552988613 | 66 | 1.1690881674 |
| 55 | 0.984375 | 1.2645766677 | 63 | 1.2062210161 |
| 56 | 0.9921875 | 1.2741490985 | 67 | 1.2445189251 |
| 57 | 1.0 | 1.2840254234 | 68 | 1.2840254234 |
| 58 | 1.0078125 | 1.2942152614 | 68 | 1.3247860267 |
| 59 | 1.015625 | 1.3047285945 | 66 | 1.3668483362 |
| 60 | 1.0234375 | 1.3155757860 | 69 | 1.4102621481 |
| 61 | 1.03125 | 1.3267675959 | 70 | 1.4550795621 |
| 62 | 1.0390625 | 1.3383152012 | 70 | 1.5013551044 |
| 63 | 1.046875 | 1.3502302147 | 68 | 1.5491458513 |
| 64 | 1.0546875 | 1.3625247063 | 72 | 1.5985115644 |
| 65 | 1.0625 | 1.3752112224 | 72 | 1.6495148280 |
| 66 | 1.0703125 | 1.3883028104 | 73 | 1.7022212012 |
| 67 | 1.078125 | 1.4018130421 | 71 | 1.7566993738 |
| 68 | 1.0859375 | 1.4157560394 | 74 | 1.8130213350 |
| 69 | 1.09375 | 1.4301464985 | 75 | 1.8712625465 |

Table 1 ($y' = x^3y$, five nodes, 4 times) 3/5

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{10}$ | | |
|-----------------|-----------|---|-----|--------------|
| S | x | y | E | y' |
| 70 | 1.1015625 | 1.4449997204 | 76 | 1.9315021331 |
| 71 | 1.109375 | 1.4603316389 | 74 | 1.9938230789 |
| 72 | 1.1171875 | 1.4761588526 | 77 | 2.0583124399 |
| 73 | 1.125 | 1.4924986555 | 78 | 2.1250615622 |
| 74 | 1.1328125 | 1.5093690736 | 79 | 2.1941663227 |
| 75 | 1.140625 | 1.5267889001 | 77 | 2.2657273771 |
| 76 | 1.1484375 | 1.5447777352 | 81 | 2.3398504285 |
| 77 | 1.15625 | 1.5633560239 | 82 | 2.4166465050 |
| 78 | 1.1640625 | 1.5825451012 | 83 | 2.4962322640 |
| 79 | 1.171875 | 1.6023672362 | 81 | 2.5787303077 |
| 80 | 1.1796875 | 1.6228456814 | 85 | 2.6642695243 |
| 81 | 1.1875 | 1.6440047199 | 86 | 2.7529854428 |
| 82 | 1.1953125 | 1.6658697222 | 87 | 2.8450206193 |
| 83 | 1.203125 | 1.6884672006 | 86 | 2.9405250415 |
| 84 | 1.2109375 | 1.7118248708 | 90 | 3.0396565631 |
| 85 | 1.21875 | 1.7359717122 | 91 | 3.1425813598 |

the first approximate value of $y_{83.5}$ for $x = 1.20703125$

1.7001460357 by 2_{-1}^{+1} 1.7000473893 by 3_{-1}^{+2}

1.7000492005 by 4_{-2}^{+2} 1.7000492301 by 5_{-3}^{+2}

1.7000492308 by 6_{-4}^{+2} 1.7000492308 by 7_{-5}^{+2}

1.7000492308 by 8_{-6}^{+2} 1.7000492307 by 9_{-7}^{+2}

the first approximate value of $y_{84.5}$ for $x = 1.21484375$

1.7238982915 by 2_{-1}^{+1} 1.7237996451 by 3_{-2}^{+1}

1.7237978339 by 4_{-3}^{+1} 1.7237977845 by 5_{-4}^{+1}

1.7237977829 by 6_{-5}^{+1} 1.7237977829 by 7_{-6}^{+1}

1.7237977830 by 8_{-7}^{+1} 1.7237977832 by 9_{-8}^{+1}

| | | | | |
|-----|------------|--------------|-----|--------------|
| 86 | 1.22265625 | 1.7483504910 | 92 | 3.1955205628 |
| 87 | 1.2265625 | 1.7609380377 | 92 | 3.2494744233 |
| 88 | 1.23046875 | 1.7737383614 | 93 | 3.3044662778 |
| 89 | 1.234375 | 1.7867555633 | 94 | 3.3605200812 |
| 90 | 1.23828125 | 1.7999938393 | 94 | 3.4176604242 |
| 91 | 1.2421875 | 1.8134574829 | 95 | 3.4759125526 |
| 92 | 1.24609375 | 1.8271508875 | 96 | 3.5353023866 |
| 93 | 1.25 | 1.8410785489 | 97 | 3.5958565408 |
| 94 | 1.25390625 | 1.8552450683 | 97 | 3.6576023444 |
| 95 | 1.2578125 | 1.8696551553 | 98 | 3.7205678635 |
| 96 | 1.26171875 | 1.8843136304 | 99 | 3.7847819226 |
| 97 | 1.265625 | 1.8992254283 | 100 | 3.8502741274 |
| 98 | 1.26953125 | 1.9143956008 | 100 | 3.9170748880 |
| 99 | 1.2734375 | 1.9298293203 | 101 | 3.9852154443 |
| 100 | 1.27734375 | 1.9455318828 | 102 | 4.0547278900 |
| 101 | 1.28125 | 1.9615087112 | 103 | 4.1256451991 |
| 102 | 1.28515625 | 1.9777653592 | 104 | 4.1980012525 |
| 103 | 1.2890625 | 1.9943075146 | 104 | 4.2718308659 |
| 104 | 1.29296875 | 2.0111410031 | 105 | 4.3471698186 |
| 105 | 1.296875 | 2.0282717920 | 107 | 4.4240548825 |
| 106 | 1.30078125 | 2.0457059942 | 107 | 4.5025238533 |
| 107 | 1.3046875 | 2.0634498725 | 108 | 4.5826155822 |
| 108 | 1.30859375 | 2.0815098434 | 109 | 4.6643700079 |
| 109 | 1.3125 | 2.0998924816 | 110 | 4.7478281914 |
| 110 | 1.31640625 | 2.1186045244 | 111 | 4.8330323499 |

Table 1 ($y' = x^3y$, five nodes, 4 times) 4/5

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{10}$ | | |
|-----------------|------------|---|-----|---------------|
| S | x | y | E | y' |
| 111 | 1.3203125 | 2.1376528767 | 112 | 4.9200258941 |
| 112 | 1.32421875 | 2.1570446147 | 113 | 5.0088534650 |
| 113 | 1.328125 | 2.1767869922 | 114 | 5.0995609724 |
| 114 | 1.33203125 | 2.1968874441 | 115 | 5.1921956355 |
| 115 | 1.3359375 | 2.2173535930 | 116 | 5.2868060240 |
| 116 | 1.33984375 | 2.2381932537 | 117 | 5.3834421009 |
| 117 | 1.34375 | 2.2594144390 | 119 | 5.4821552673 |
| 118 | 1.34765625 | 2.2810253651 | 120 | 5.5829984073 |
| 119 | 1.3515625 | 2.3030344582 | 121 | 5.6860259369 |
| 120 | 1.35546875 | 2.3254503598 | 122 | 5.7912938520 |
| 121 | 1.359375 | 2.3482819335 | 123 | 5.8988597797 |
| 122 | 1.36328125 | 2.3715382712 | 124 | 6.0087830309 |
| 123 | 1.3671875 | 2.3952286998 | 126 | 6.1211246552 |
| 124 | 1.37109375 | 2.4193627882 | 127 | 6.2359474969 |
| 125 | 1.375 | 2.4439503546 | 128 | 6.3533162537 |
| 126 | 1.37890625 | 2.4690014731 | 129 | 6.4732975370 |
| 127 | 1.3828125 | 2.4945264825 | 131 | 6.5959599352 |
| 128 | 1.38671875 | 2.5205359930 | 132 | 6.7213740781 |
| 129 | 1.390625 | 2.5470408951 | 134 | 6.8496127044 |
| 130 | 1.39453125 | 2.5740523674 | 135 | 6.9807507314 |
| 131 | 1.3984375 | 2.6015818858 | 136 | 7.1148653275 |
| 132 | 1.40234375 | 2.6296412319 | 138 | 7.2520359868 |
| 133 | 1.40625 | 2.6582425028 | 140 | 7.3923446065 |
| 134 | 1.41015625 | 2.6873981198 | 141 | 7.5358755675 |
| 135 | 1.4140625 | 2.7171208395 | 142 | 7.6827158183 |
| 136 | 1.41796875 | 2.7474237627 | 144 | 7.8329549603 |
| 137 | 1.421875 | 2.7783203459 | 146 | 7.9866853385 |
| 138 | 1.42578125 | 2.8098244116 | 147 | 8.1440021330 |
| 139 | 1.4296875 | 2.8419501599 | 149 | 8.3050034570 |
| 140 | 1.43359375 | 2.8747121802 | 151 | 8.4697904554 |
| 141 | 1.4375 | 2.9081254628 | 153 | 8.6384674087 |
| 142 | 1.44140625 | 2.9422054117 | 154 | 8.8111418405 |
| 143 | 1.4453125 | 2.9769678577 | 156 | 8.9879246292 |
| 144 | 1.44921875 | 3.0124290711 | 158 | 9.1689301231 |
| 145 | 1.453125 | 3.0486057760 | 160 | 9.3542762611 |
| 146 | 1.45703125 | 3.0855151645 | 162 | 9.5440846959 |
| 147 | 1.4609375 | 3.1231749112 | 164 | 9.7384809250 |
| 148 | 1.46484375 | 3.1616031888 | 166 | 9.9375944232 |
| 149 | 1.46875 | 3.2008186837 | 168 | 10.1415587829 |
| 150 | 1.47265625 | 3.2408406123 | 170 | 10.3505118574 |
| 151 | 1.4765625 | 3.2816887383 | 172 | 10.5645959123 |
| 152 | 1.48046875 | 3.3233833898 | 174 | 10.7839577804 |
| 153 | 1.484375 | 3.3659454778 | 177 | 11.0087490235 |
| 154 | 1.48828125 | 3.4093965146 | 179 | 11.2391261005 |
| 155 | 1.4921875 | 3.4537586337 | 181 | 11.4752505422 |
| 156 | 1.49609375 | 3.4990546099 | 183 | 11.7172891320 |
| 157 | 1.5 | 3.5453078798 | 186 | 11.9654140944 |
| 158 | 1.50390625 | 3.5925425638 | 188 | 12.2198032908 |
| 159 | 1.5078125 | 3.6407834885 | 191 | 12.4806404226 |
| 160 | 1.51171875 | 3.6900562097 | 193 | 12.7481152426 |
| 161 | 1.515625 | 3.7403870366 | 196 | 13.0224237743 |
| 162 | 1.51953125 | 3.7918030569 | 199 | 13.3037685403 |
| 163 | 1.5234375 | 3.8443321623 | 202 | 13.5923587998 |

Table 1 ($y' = x^3y$, five nodes, 4 times) 5/5

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{10}$ | | |
|-----------------|------------|---|-----|---------------|
| S | x | y | E | y' |
| 164 | 1.52734375 | 3.8980030755 | 204 | 13.8884107948 |
| 165 | 1.53125 | 3.9528453781 | 208 | 14.1921480068 |
| 166 | 1.53515625 | 4.0088895390 | 210 | 14.5038014235 |
| 167 | 1.5390625 | 4.0661669448 | 213 | 14.8236098162 |
| 168 | 1.54296875 | 4.1247099302 | 216 | 15.1518200285 |
| 169 | 1.546875 | 4.1845518106 | 220 | 15.4886872760 |
| 170 | 1.55078125 | 4.2457269150 | 223 | 15.8344754587 |
| 171 | 1.5546875 | 4.3082706208 | 226 | 16.1894574863 |
| 172 | 1.55859375 | 4.3722193898 | 229 | 16.5539156159 |
| 173 | 1.5625 | 4.4376108050 | 233 | 16.9281418038 |
| 174 | 1.56640625 | 4.5044836093 | 236 | 17.3124380716 |
| 175 | 1.5703125 | 4.5728777459 | 240 | 17.7071168881 |
| 176 | 1.57421875 | 4.6428343994 | 243 | 18.1125015656 |
| 177 | 1.578125 | 4.7143960395 | 247 | 18.5289266733 |
| 178 | 1.58203125 | 4.7876064650 | 251 | 18.9567384671 |
| 179 | 1.5859375 | 2.8625108512 | 255 | 19.3962953389 |
| 180 | 1.58984375 | 4.9391557979 | 259 | 19.8479682824 |
| 181 | 1.59375 | 5.0175893791 | 263 | 20.3121413797 |
| 182 | 1.59765625 | 5.0978611958 | 267 | 20.7892123072 |
| 183 | 1.6015625 | 5.1800224298 | 272 | 21.2795928649 |
| 184 | 1.60546875 | 5.2641259000 | 276 | 21.7837095250 |
| 185 | 1.609375 | 5.3502261206 | 281 | 22.3020040056 |
| 186 | 1.61328125 | 5.4383793620 | 285 | 22.8349338677 |

the first approximate value of $y_{184.5}$ for $x_{184.5} = 1.607421875$

5.3071760103 by 2^{+1}_{-1} 5.3069193827 by 3^{+2}_{-1}
5.3069228996 by 4^{+2}_{-2} 5.3069229407 by 5^{+2}_{-3}
5.3069229414 by 6^{+2}_{-4} 5.3069229414 by 7^{+2}_{-5}
5.3069229414 by 8^{+2}_{-6} 5.3069229414 by 9^{+2}_{-7}

the first approximate value of $y_{185.5}$ for $x_{185.5} = 1.611328125$

5.3943027413 by 2^{+1}_{-1} 5.3940461137 by 3^{+1}_{-2}
5.3940425968 by 4^{+1}_{-3} 5.3940425282 by 5^{+1}_{-4}
5.3940425265 by 6^{+1}_{-5} 5.3940425265 by 7^{+2}_{-5}
5.3940425265 by 8^{+1}_{-6} 5.3940425265 by 9^{+1}_{-8}

| | | | | |
|-----|-------------|--------------|-----|---------------|
| 187 | 1.615234375 | 5.4832439459 | 288 | 23.1070345399 |
| 188 | 1.6171875 | 5.5286437144 | 290 | 23.3829731393 |
| 189 | 1.619140625 | 5.5745862237 | 293 | 23.6628115988 |
| 190 | 1.62109375 | 5.6210791521 | 295 | 23.9466129628 |
| 191 | 1.623046875 | 5.6681303023 | 297 | 24.2344414100 |
| 192 | 1.625 | 5.7157476032 | 300 | 24.5263622740 |
| 193 | 1.626953125 | 5.7639391126 | 303 | 24.8224420679 |
| 194 | 1.62890625 | 5.8127130192 | 305 | 25.1227485062 |
| 195 | 1.630859375 | 5.8620776455 | 308 | 25.4273505296 |
| 196 | 1.6328125 | 5.9120414495 | 310 | 25.7363183281 |
| 197 | 1.634765625 | 5.9626130278 | 313 | 26.0497233668 |
| 198 | 1.63671875 | 6.0138011176 | 315 | 26.3676384105 |
| 199 | 1.638671875 | 6.0656145997 | 318 | 26.6901375504 |
| 200 | 1.640625 | 6.1180625009 | 321 | 27.0172962290 |
| 205 | 1.650390625 | 6.3901447921 | 335 | 28.7257213089 |

Table 2 ($y' = x^3 y$, five nodes, 5 times) 1/5

| S | x | y | E | y' |
|-----|---------|--------------|-----|---------------|
| 1 | -0.125 | 1.0000610370 | 0 | -0.0019532442 |
| 2 | -0.0625 | 1.0000038147 | 0 | -0.0002441416 |
| 3 | 0 | 1.0000000000 | 0 | 0.0000000000 |
| 4 | 0.0625 | 1.0000038147 | 0 | 0.0002441416 |
| 5 | 0.125 | 1.0000610370 | 0 | 0.0019532442 |
| 6 | 0.1875 | 1.0003090407 | 25 | 0.0065938340 |
| 7 | 0.25 | 1.0009770445 | 50 | 0.0156402663 |
| 8 | 0.3125 | 1.0023870378 | 76 | 0.0305904247 |
| 9 | 0.375 | 1.0049560990 | 103 | 0.0529957318 |
| 10 | 0.4375 | 1.0092011770 | 161 | 0.0845107431 |
| 11 | 0.5 | 1.0157477314 | 228 | 0.1269684664 |

the first approximate value of $y_{9.5}$ for $x_{9.5} = 0.40625$ 1.0070786380 by 2^{+1}_{-1} 1.0067909535 by 3^{+2}_{-1} 1.0068300447 by 4^{+2}_{-2} 1.0068325879 by 5^{+2}_{-3} 1.0068326891 by 6^{+2}_{-4} 1.0068327155 by 7^{+2}_{-5} 1.0068327217 by 8^{+2}_{-6} 1.0068327283 by 9^{+2}_{-7} the first approximate value of $y_{10.5}$ for $x_{10.5} = 0.46875$ 1.0124744542 by 2^{+1}_{-1} 1.0121867697 by 3^{+1}_{-2} 1.0121476784 by 4^{+1}_{-3} 1.0121434398 by 5^{+1}_{-4} 1.0121432035 by 6^{+1}_{-5} 1.0121431244 by 7^{+1}_{-6} 1.0121431019 by 8^{+1}_{-7} 1.0121430914 by 9^{+1}_{-8}

| | | | | |
|----|---------|--------------|-----|--------------|
| 12 | 0.53125 | 1.0201125592 | 156 | 0.1529483949 |
| 13 | 0.5625 | 1.0253440809 | 166 | 0.1824892175 |
| 14 | 0.59375 | 1.0315587038 | 158 | 0.2159259384 |
| 15 | 0.625 | 1.0388839327 | 235 | 0.2536337726 |
| 16 | 0.65625 | 1.0474596999 | 165 | 0.2960365076 |
| 17 | 0.6875 | 1.0574401127 | 176 | 0.3436164038 |
| 18 | 0.71875 | 1.0689953793 | 171 | 0.3969258661 |
| 19 | 0.75 | 1.0823142640 | 249 | 0.4566013301 |
| 20 | 0.78125 | 1.0976069107 | 183 | 0.5233797601 |

the first approximate value of $y_{18.5}$ for $x_{18.5} = 0.734375$ 1.0756548216 by 2^{+1}_{-1} 1.0754081014 by 3^{+2}_{-1} 1.0754212354 by 4^{+2}_{-2} 1.0754217364 by 5^{+2}_{-3} 1.0754217695 by 6^{+2}_{-4} 1.0754217720 by 7^{+2}_{-5} 1.0754217713 by 8^{+2}_{-6} 1.0754217698 by 9^{+2}_{-7} the first approximate value of $y_{19.5}$ for $x_{19.5} = 0.765625$ 1.0899605873 by 2^{+1}_{-1} 1.0897138671 by 3^{+1}_{-2} 1.0897007331 by 4^{+1}_{-3} 1.0896998979 by 5^{+1}_{-4} 1.0896998207 by 6^{+1}_{-5} 1.0896998132 by 7^{+1}_{-6} 1.0896998158 by 8^{+1}_{-7} 1.0896998225 by 9^{+1}_{-8}

| S | x | y | E | y' |
|-----|----------|--------------|-----|--------------|
| 21 | 0.796875 | 1.1060657762 | 253 | 0.5596951724 |
| 22 | 0.8125 | 1.1151083685 | 257 | 0.5981184291 |
| 23 | 0.828125 | 1.1247686299 | 258 | 0.6387793706 |
| 24 | 0.84375 | 1.1350826015 | 191 | 0.6818185683 |
| 25 | 0.859375 | 1.1460886382 | 262 | 0.7273883712 |
| 26 | 0.875 | 1.1578275334 | 266 | 0.7756539921 |
| 27 | 0.890625 | 1.1703427976 | 267 | 0.8267947911 |
| 28 | 0.90625 | 1.1836808504 | 202 | 0.8810056232 |
| 29 | 0.921875 | 1.1978913135 | 273 | 0.9384983828 |

Table 2 ($y' = x^3 y$, five nodes, 5 times) 2/5

| S | x | y | E | y' |
|-----|----------|--------------|-----|--------------|
| 30 | 0.9375 | 1.2130272226 | 279 | 0.9995036319 |
| 31 | 0.953125 | 1.2291454003 | 280 | 1.0642725071 |
| 32 | 0.96875 | 1.2463067551 | 216 | 1.1330787518 |
| 33 | 0.984375 | 1.2645766901 | 287 | 1.0262210374 |
| 34 | 1.0 | 1.2840254462 | 295 | 1.2840254462 |
| 35 | 1.015625 | 1.3047286176 | 296 | 1.3668483604 |
| 36 | 1.03125 | 1.3267676123 | 234 | 1.4550795802 |
| 37 | 1.046875 | 1.3502302384 | 305 | 1.5491458786 |
| 38 | 1.0625 | 1.3752112467 | 316 | 1.6495148572 |
| 39 | 1.078125 | 1.4018130668 | 318 | 1.7566994047 |
| 40 | 1.09375 | 1.4301465167 | 258 | 1.8712625704 |
| 41 | 1.109375 | 1.4603316644 | 329 | 1.9938231138 |
| 42 | 1.125 | 1.4924986818 | 342 | 2.1250615997 |

the first approximate value of $y_{40.5}$ for $x_{40.5} = 1.1015625$ 1.4452390906 by 2^{+1}_{-1} 1.4449913569 by 3^{+2}_{-1} 1.4449994926 by 4^{+2}_{-2} 1.4449997294 by 5^{+2}_{-3} 1.4449997394 by 6^{+2}_{-4} 1.4449997394 by 7^{+2}_{-5} 1.4449997387 by 8^{+2}_{-6} 1.4449997377 by 9^{+2}_{-7} the first approximate value of $y_{41.5}$ for $x_{41.5} = 1.1171875$ 1.4764151731 by 2^{+1}_{-1} 1.4761674394 by 3^{+1}_{-2} 1.4761593037 by 4^{+1}_{-3} 1.4761589090 by 5^{+1}_{-4} 1.4761588856 by 6^{+1}_{-5} 1.4761588858 by 7^{+1}_{-6} 1.4761588883 by 8^{+1}_{-7} 1.4761588928 by 9^{+1}_{-8}

| | | | | |
|----|-----------|--------------|-----|--------------|
| 43 | 1.1328125 | 1.5093690997 | 340 | 2.1941663606 |
| 44 | 1.140625 | 1.5267889268 | 344 | 2.2657274168 |
| 45 | 1.1484375 | 1.5447777620 | 348 | 2.3398504690 |
| 46 | 1.15625 | 1.5633560515 | 358 | 2.4166465477 |
| 47 | 1.1640625 | 1.5825451286 | 357 | 2.4962323072 |
| 48 | 1.171875 | 1.6023672643 | 362 | 2.5787303528 |
| 49 | 1.1796875 | 1.6228457095 | 366 | 2.6642695705 |
| 50 | 1.1875 | 1.6440047489 | 377 | 2.7529854914 |
| 51 | 1.1953125 | 1.6658697510 | 376 | 2.8450206686 |
| 52 | 1.203125 | 1.6884672301 | 381 | 2.9405250929 |
| 53 | 1.2109375 | 1.7118249004 | 386 | 3.0396566158 |
| 54 | 1.21875 | 1.7359717429 | 398 | 3.1425814153 |
| 55 | 1.2265625 | 1.7609380683 | 398 | 3.2494744796 |
| 56 | 1.234375 | 1.7867555942 | 403 | 3.3605201394 |
| 57 | 1.2421875 | 1.8134575144 | 409 | 3.4759126129 |
| 58 | 1.25 | 1.8410785813 | 421 | 3.5958566042 |
| 59 | 1.2578125 | 1.8696551877 | 422 | 3.7205679281 |
| 60 | 1.265625 | 1.8992254612 | 429 | 3.8502741941 |
| 61 | 1.2734275 | 1.9298293538 | 436 | 3.9852155135 |
| 62 | 1.28125 | 1.9615087458 | 449 | 4.1256452718 |
| 63 | 1.2890625 | 1.9943075492 | 451 | 4.2718309400 |
| 64 | 1.296875 | 2.0282718271 | 458 | 4.4240549592 |
| 65 | 1.3046875 | 2.0634499083 | 466 | 4.5826156617 |
| 66 | 1.3125 | 2.0998925185 | 480 | 4.7478282750 |
| 67 | 1.3203125 | 2.1376529138 | 483 | 4.9200259795 |
| 68 | 1.328125 | 2.1767870299 | 492 | 5.0995610608 |
| 69 | 1.3359375 | 2.2173536315 | 501 | 5.2868061157 |
| 70 | 1.34375 | 2.2594144787 | 516 | 5.4821553637 |
| 71 | 1.3515625 | 2.3030344982 | 521 | 5.6860260357 |

Table 2 ($y' = x^3y$, five nodes, 5 times) 3/5 $S=\text{step}$ $E = (\text{error of } y) \times 10^{10}$

| S | x | y | E | y' |
|-----|-----------|--------------|-----|---------------|
| 72 | 1.359375 | 2.3482819742 | 531 | 5.8988598820 |
| 73 | 1.3671875 | 2.3952287414 | 542 | 6.1211247615 |
| 74 | 1.375 | 2.4439503975 | 558 | 6.3533163654 |
| 75 | 1.3828125 | 2.4945265258 | 564 | 6.5959600498 |
| 76 | 1.390625 | 2.5470409393 | 576 | 6.8496128232 |
| 77 | 1.3984375 | 2.6015819310 | 588 | 7.1148654512 |
| 78 | 1.40625 | 2.6582425495 | 607 | 7.3923447363 |
| 79 | 1.4140625 | 2.7171208867 | 615 | 7.6827159519 |
| 80 | 1.421875 | 2.7783203941 | 628 | 7.9866854771 |
| 81 | 1.4296875 | 2.8419502094 | 643 | 8.3050036014 |
| 82 | 1.4375 | 2.9081255139 | 663 | 8.6384675604 |
| 83 | 1.4453125 | 2.9769679095 | 674 | 8.9879247855 |
| 84 | 1.453125 | 3.0486058289 | 690 | 9.3542764235 |
| 85 | 1.4609375 | 3.1231749655 | 707 | 9.7384810944 |
| 86 | 1.46875 | 3.2008187399 | 730 | 10.1415589609 |
| 87 | 1.4765625 | 3.2816887955 | 743 | 10.5645960962 |
| 88 | 1.484375 | 3.3659455363 | 762 | 11.0087492148 |
| 89 | 1.4921875 | 3.4537586939 | 782 | 11.4752507420 |
| 90 | 1.5 | 3.5453079421 | 808 | 11.9654143044 |
| 91 | 1.5078125 | 3.6407835519 | 825 | 12.4806406400 |
| 92 | 1.515625 | 3.7403871017 | 847 | 13.0224240008 |
| 93 | 1.5234375 | 3.8443322293 | 871 | 13.5923590367 |

the first value of $y_{91.5}$ for $x_{91.5}=1.51171875$ 3.6905853268 by 2_{-1}^{+1} 3.6900426296 by 3_{-1}^{+2} 3.6900559820 by 4_{-2}^{+2} 3.6900562646 by 5_{-3}^{+2} 3.6900562734 by 6_{-4}^{+2} 3.6900562737 by 7_{-5}^{+2} 3.6900562737 by 8_{-6}^{+2} 3.6900562736 by 9_{-7}^{+2} the first value of $y_{92.5}$ for $x_{92.5}=1.51953125$ 3.7923596655 by 2_{-1}^{+1} 3.7918169683 by 3_{-2}^{+1} 3.7918036159 by 4_{-3}^{+1} 3.7918031448 by 5_{-4}^{+1} 3.7918031242 by 6_{-5}^{+1} 3.7918031233 by 7_{-6}^{+1} 3.7918031235 by 8_{-7}^{+1} 3.7918031239 by 9_{-8}^{+1}

| | | | | |
|-----|------------|--------------|------|---------------|
| 94 | 1.52734375 | 3.8980031433 | 883 | 13.8884110364 |
| 95 | 1.53125 | 3.9528454469 | 895 | 14.1921482537 |
| 96 | 1.53515625 | 4.0088896088 | 908 | 14.5038016758 |
| 97 | 1.5390625 | 4.0661670157 | 922 | 14.8236100745 |
| 98 | 1.54296875 | 4.1247100020 | 934 | 15.1518202921 |
| 99 | 1.546875 | 4.1845518834 | 948 | 15.4886875454 |
| 100 | 1.55078125 | 4.2457269888 | 961 | 15.8344757342 |
| 101 | 1.5546875 | 4.3082706959 | 977 | 16.1894577684 |
| 102 | 1.55859375 | 4.3722194659 | 990 | 16.5539159039 |
| 103 | 1.5625 | 4.4376108822 | 1005 | 16.9281420983 |
| 104 | 1.56640625 | 4.5044836877 | 1020 | 17.3124383728 |
| 105 | 1.5703125 | 4.5728778256 | 1036 | 17.7071171966 |
| 106 | 1.57421875 | 4.6428344802 | 1051 | 18.1125018808 |
| 107 | 1.578125 | 4.7143961215 | 1068 | 18.5289269956 |
| 108 | 1.58203125 | 4.7876065483 | 1084 | 18.9567387970 |
| 109 | 1.5859375 | 4.8625109359 | 1102 | 19.3962956769 |
| 110 | 1.58984375 | 4.9391558838 | 1118 | 19.8479686278 |
| 111 | 1.59375 | 5.0175894664 | 1136 | 20.3121417330 |
| 112 | 1.59765625 | 5.0978612845 | 1154 | 20.7892126690 |
| 113 | 1.6015625 | 5.1800225201 | 1174 | 21.2795932357 |

Table 2 ($y' = x^3y$, five nodes, 5 times) 4/5 $S=\text{step}$ $E=(\text{error of } y) \times 10^{10}$

| S | x | y | E | y' |
|-----|------------|---------------|------|---------------|
| 114 | 1.60546875 | 5.2641259916 | 1192 | 21.7837099041 |
| 115 | 1.609375 | 5.3502262136 | 1212 | 22.3020043935 |
| 116 | 1.61328125 | 5.4383794567 | 1232 | 22.8349342650 |
| 117 | 1.6171875 | 5.5286438106 | 1253 | 23.3829735464 |
| 118 | 1.62109375 | 5.6210792500 | 1273 | 23.9466133795 |
| 119 | 1.625 | 5.7157477027 | 1294 | 24.5263627007 |
| 120 | 1.62890625 | 5.8127131204 | 1316 | 25.1227489433 |
| 121 | 1.6328125 | 5.9120415525 | 1340 | 25.7363187763 |
| 122 | 1.63671875 | 6.1308012222 | 1362 | 26.3676388694 |
| 123 | 1.640625 | 6.1180626074 | 1385 | 27.0172966990 |
| 124 | 1.64453125 | 6.2248985221 | 1410 | 27.6859013796 |
| 125 | 1.6484375 | 6.3343842038 | 1435 | 28.3740845384 |
| 126 | 1.65234375 | 6.4465974022 | 1460 | 29.0825012234 |
| 127 | 1.65625 | 6.5616184738 | 1486 | 29.8118308571 |
| 128 | 1.66015625 | 6.6796304777 | 1513 | 30.5627782244 |
| 129 | 1.6640625 | 6.8004192777 | 1541 | 31.3360745102 |
| 130 | 1.66796875 | 6.9243736466 | 1568 | 32.1324783776 |
| 131 | 1.671875 | 7.0514853768 | 1597 | 32.9527771012 |
| 132 | 1.67578125 | 7.1818493933 | 1626 | 33.7977877416 |
| 133 | 1.6796875 | 7.3155638735 | 1658 | 34.6683583788 |
| 134 | 1.68359375 | 7.4527303693 | 1688 | 35.5653693964 |
| 135 | 1.6875 | 7.5934539380 | 1720 | 36.4897348293 |
| 136 | 1.69140625 | 7.7378432746 | 1752 | 37.4424037640 |
| 137 | 1.6953125 | 7.8860108527 | 1787 | 38.4243618079 |
| 138 | 1.69921875 | 8.0380730692 | 1820 | 39.4366326207 |
| 139 | 1.703125 | 8.1941503974 | 1856 | 40.4802795220 |
| 140 | 1.70703125 | 8.3543675438 | 1892 | 41.5564071631 |
| 141 | 1.7109375 | 8.5188536136 | 1930 | 42.6661632813 |
| 142 | 1.71484375 | 8.6877422824 | 1968 | 43.8107405304 |
| 143 | 1.71875 | 8.8611719757 | 2007 | 44.9913784015 |
| 144 | 1.72265625 | 9.0392860554 | 2047 | 46.2093652237 |
| 145 | 1.7265625 | 9.2222330146 | 2090 | 47.4660402629 |
| 146 | 1.73046875 | 9.4101666808 | 2131 | 48.7627959144 |
| 147 | 1.734375 | 9.6032464286 | 2175 | 50.1010800032 |
| 148 | 1.73828125 | 9.8016373997 | 2220 | 51.4823981825 |
| 149 | 1.7421875 | 10.0055107348 | 2267 | 52.9083164513 |
| 150 | 1.74609375 | 10.2150438132 | 2314 | 54.3804637854 |
| 151 | 1.75 | 10.4304205058 | 2362 | 55.9005348982 |
| 152 | 1.75390625 | 10.6518314358 | 2413 | 57.4702931231 |
| 153 | 1.7578125 | 10.8794742537 | 2465 | 59.0915734392 |
| 154 | 1.76171875 | 11.1135539227 | 2517 | 60.7662856353 |
| 155 | 1.765625 | 11.3542830178 | 2572 | 62.4964176313 |
| 156 | 1.76953125 | 11.6018820376 | 2628 | 64.2840389494 |
| 157 | 1.7734375 | 11.8565797299 | 2687 | 66.1313043579 |
| 158 | 1.77734375 | 12.1186134321 | 2745 | 68.0404576855 |
| 159 | 1.78125 | 12.3882294273 | 2806 | 70.0138358256 |
| 160 | 1.78515625 | 12.6656833156 | 2869 | 72.0538729258 |
| 161 | 1.7890625 | 12.9512404024 | 2935 | 74.1631047862 |
| 162 | 1.79296875 | 13.2451761044 | 3000 | 76.3441734676 |
| 163 | 1.796875 | 13.5477763748 | 3069 | 78.5993213074 |
| 164 | 1.80078125 | 13.8593381462 | 3140 | 80.9329501031 |
| 165 | 1.8046875 | 14.1801697949 | 3213 | 83.3465182009 |
| 166 | 1.80859375 | 14.5105916264 | 3287 | 85.8436543090 |
| 167 | 1.8125 | 14.8509363834 | 3364 | 88.4276092418 |

Table 2 ($y' = x^3y$, five nodes, 5 times) 5/5

| S | x | y | E | y' |
|-----|------------|---------------|------|----------------|
| 168 | 1.81640625 | 15.2015497765 | 3444 | 91.1017728864 |
| 169 | 1.8203125 | 15.5627910402 | 3526 | 93.8696806566 |
| 170 | 1.82421875 | 15.9350335141 | 3610 | 96.7350202641 |
| 171 | 1.828125 | 16.3186652523 | 3697 | 99.7016388351 |
| 172 | 1.83203125 | 16.7140896601 | 3786 | 102.7735503745 |
| 173 | 1.8359375 | 17.1217261611 | 3880 | 105.9549436107 |
| 174 | 1.83984375 | 17.5420108954 | 3974 | 109.2501902336 |
| 175 | 1.84375 | 17.9753974519 | 4072 | 112.6638535546 |
| 176 | 1.84765625 | 18.4223576337 | 4174 | 116.2006975973 |
| 177 | 1.8515625 | 18.8833822610 | 4279 | 119.8656966577 |
| 178 | 1.85546875 | 19.3589820112 | 4386 | 123.6640453477 |
| 179 | 1.859375 | 19.8496883014 | 4497 | 127.6011691602 |
| 180 | 1.86328125 | 20.3560542106 | 4612 | 131.6827355678 |
| 181 | 1.8671875 | 20.8786554477 | 4731 | 135.9146656995 |
| 182 | 1.87109375 | 21.4180913664 | 4853 | 140.3031466172 |
| 183 | 1.875 | 21.9749860292 | 4979 | 144.8546442355 |
| 184 | 1.87890625 | 22.5499893225 | 5109 | 149.5759169046 |
| 185 | 1.8828125 | 23.1437781268 | 5245 | 154.4740297073 |
| 186 | 1.88671875 | 23.7570575438 | 5383 | 159.5563695023 |
| 187 | 1.890625 | 24.3905621848 | 5526 | 164.8306607618 |
| 188 | 1.89453125 | 25.0450575209 | 5675 | 170.3049822360 |

the first value of $y_{186.5}$ for $x_{186.5}=1.888671875$

24.0738098643 by 2^{+1}_{-1} 24.0711860275 by 3^{+2}_{-1}
 24.0712338694 by 4^{+2}_{-2} 24.0712345927 by 5^{+2}_{-3}
 24.0712346085 by 6^{+2}_{-4} 24.0712346089 by 7^{+2}_{-5}
 24.0712346089 by 8^{+2}_{-6} 24.0712346089 by 9^{+2}_{-7}

the first value of $y_{187.5}$ for $x_{187.5}=1.892578125$

24.7178098529 by 2^{+1}_{-1} 24.7151860160 by 3^{+1}_{-2}
 24.7151381741 by 4^{+1}_{-3} 24.7151369686 by 5^{+1}_{-4}
 24.7151369317 by 6^{+1}_{-5} 24.7151369304 by 7^{+2}_{-5}
 24.7151369304 by 8^{+1}_{-6} 24.7151369304 by 9^{+1}_{-8}

| | | | | |
|-----|-------------|---------------|------|----------------|
| 189 | 1.896484375 | 25.3804246951 | 5751 | 173.1197800853 |
| 190 | 1.8984375 | 25.7213413010 | 5828 | 175.9877845017 |
| 191 | 1.900390625 | 25.0679123449 | 5906 | 178.9101129364 |
| 192 | 1.90234375 | 26.4202450405 | 5987 | 181.8879084442 |
| 193 | 1.904296875 | 26.7784488593 | 6067 | 184.9223403127 |
| 194 | 1.90625 | 27.1426355831 | 6150 | 188.0146047146 |
| 195 | 1.908203125 | 27.5129193564 | 6234 | 191.1659253700 |
| 196 | 1.91015625 | 27.8894167416 | 6319 | 194.3775542325 |
| 197 | 1.912109375 | 28.2722467741 | 6406 | 197.6507721881 |
| 198 | 1.9140625 | 28.6615310201 | 6494 | 200.9868897785 |
| 199 | 1.916015625 | 29.0573936349 | 6584 | 204.3872479376 |
| 200 | 1.91796875 | 29.4599614233 | 6675 | 207.8532187522 |
| 201 | 1.919921875 | 29.8693639006 | 6768 | 211.3862062393 |
| 202 | 1.921875 | 30.2857333566 | 6862 | 214.9876471488 |
| 203 | 1.923828125 | 30.7092049198 | 6958 | 218.6590117822 |
| 204 | 1.92578125 | 31.1399166236 | 7056 | 222.4018048377 |
| 205 | 1.927734375 | 31.5780094746 | 7155 | 226.2175662741 |

Table 3 ($y' = x^3y$, seven nodes, 4 times) 1/6

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{12}$ | | |
|-----------------|---------|---|-----|-----------------|
| S | x | y | E | y' |
| 1 | -0.1875 | 1.000309038221 | 0 | -0.006593833992 |
| 2 | -0.125 | 1.000061037019 | 0 | -0.001953244213 |
| 3 | -0.0625 | 1.000003814705 | 0 | -0.000244141556 |
| 4 | 0 | 1.000000000000 | 0 | 0.000000000000 |
| 5 | 0.0625 | 1.000003814705 | 0 | 0.000244141556 |
| 6 | 0.125 | 1.000061037019 | 0 | 0.001953244213 |
| 7 | 0.1875 | 1.000309038221 | 0 | 0.006593833992 |
| 8 | 0.25 | 1.000977039504 | 12 | 0.015640266242 |
| 9 | 0.3125 | 1.002387030274 | 52 | 0.030590424508 |
| 10 | 0.375 | 1.004956088785 | 150 | 0.052995731245 |

the first approximate value of $y_{6.5}$ for $x_{6.5}=0.15625$ 1.0001850376 by 2_{-1}^{+1} 1.0001489475 by 5_{-1}^{+4} 1.0001490110 by 6_{-2}^{+4} 1.0001490206 by 7_{-3}^{+4} 1.0001490224 by 8_{-4}^{+4} 1.0001490227 by 9_{-5}^{+4} the first approximate value of $y_{7.5}$ for $x_{7.5}=0.21875$ 1.0006430389 by 2_{-1}^{+1} 1.0005726432 by 5_{-2}^{+3} 1.0005726160 by 6_{-3}^{+3} 1.0005726091 by 7_{-4}^{+3} 1.0005726073 by 8_{-5}^{+3} 1.0005726069 by 9_{-6}^{+3} the first approximate value of $y_{8.5}$ for $x_{8.5}=0.28125$ 1.0016820349 by 2_{-1}^{+1} 1.0015654475 by 5_{-2}^{+3} 1.0015654747 by 6_{-3}^{+4} 1.0015654842 by 7_{-4}^{+2} 1.0015654875 by 8_{-5}^{+2} 1.0015654884 by 9_{-6}^{+2} the first approximate value of $y_{9.5}$ for $x_{9.5}=0.34375$ 1.0036715595 by 2_{-1}^{+1} 1.0039978331 by 5_{-4}^{+1} 1.0034968308 by 6_{-5}^{+1} 1.0034968020 by 7_{-6}^{+1} 1.0034967901 by 8_{-7}^{+1} 1.0034967862 by 9_{-8}^{+1}

| | | | | |
|----|---------|----------------|-----|----------------|
| 11 | 0.40625 | 1.006832710229 | 15 | 0.067505232677 |
| 12 | 0.4375 | 1.009201160947 | 13 | 0.084510741749 |
| 13 | 0.46875 | 1.012143076268 | 16 | 0.104247524487 |
| 14 | 0.5 | 1.015747708642 | 55 | 0.126968463580 |
| 15 | 0.53125 | 1.020112543704 | 30 | 0.152948392554 |
| 16 | 0.5625 | 1.025344064463 | 155 | 0.182489214598 |
| 17 | 0.59375 | 1.031558687975 | 23 | 0.215925935084 |
| 18 | 0.625 | 1.038883909208 | 22 | 0.253633766896 |
| 19 | 0.65625 | 1.047459683438 | 28 | 0.296036502940 |
| 20 | 0.6875 | 1.057440095159 | 68 | 0.343616398110 |

the first value of $y_{16.5}$ for $x_{16.5}=0.578125$ 1.0284513762 by 2_{-1}^{+1} 1.0283207468 by 5_{-1}^{+4} 1.0283207945 by 6_{-2}^{+4} 1.0283207944 by 7_{-3}^{+4} 1.0283207945 by 8_{-4}^{+4} 1.0283207945 by 9_{-8}^{+1} the first value of $y_{17.5}$ for $x_{17.5}=0.609375$ 1.0352212986 by 2_{-1}^{+1} 1.0350740578 by 5_{-2}^{+3} 1.0350740363 by 6_{-3}^{+3} 1.0350740369 by 7_{-4}^{+3} 1.0350740368 by 8_{-5}^{+3} 1.0350740367 by 9_{-6}^{+3} the first value of $y_{18.5}$ for $x_{18.5}=0.640625$ 1.0431717963 by 2_{-1}^{+1} 1.0430061781 by 5_{-3}^{+2} 1.0430061991 by 6_{-4}^{+2} 1.0430061996 by 7_{-5}^{+2} 1.0430061998 by 8_{-6}^{+2} 1.0430061998 by 9_{-7}^{+2} the first value of $y_{19.5}$ for $x_{19.5}=0.671875$ Table 3 ($y' = x^3y$, seven nodes, 4 times) 2/6

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{12}$ | | |
|--|-----------|---|-----|----------------|
| 1.0524498893 by 2_{-1}^{+1} | | 1.0605253783 by 5_{-4}^{+1} | | |
| 1.0522640818 by 6_{-5}^{+1} | | 1.0522640759 by 7_{-6}^{+1} | | |
| 1.0522640751 by 8_{-7}^{+1} | | 1.0522640749 by 9_{-8}^{+1} | | |
| S | x | y | E | y' |
| 21 | 0.703125 | 1.063009540236 | 23 | 0.369517304817 |
| 22 | 0.71875 | 1.068995362231 | 22 | 0.396925859749 |
| 23 | 0.734375 | 1.075421753178 | 23 | 0.425924349519 |
| 24 | 0.75 | 1.082314239057 | 29 | 0.456601319602 |
| 25 | 0.765625 | 1.089699782030 | 24 | 0.489052160858 |
| 26 | 0.78125 | 1.097606892439 | 69 | 0.523379751415 |
| 27 | 0.796875 | 1.106065750924 | 24 | 0.559695159629 |
| 28 | 0.8125 | 1.115108342803 | 23 | 0.598118415317 |
| 29 | 0.828125 | 1.124768604150 | 25 | 0.638779355927 |
| 30 | 0.84375 | 1.135082582414 | 30 | 0.681818556813 |
| 31 | 0.859375 | 1.146088612007 | 25 | 0.727388354579 |
| 32 | 0.875 | 1.157827506823 | 71 | 0.775653974298 |
| 33 | 0.890625 | 1.170342770902 | 27 | 0.826794772231 |
| 34 | 0.90625 | 1.183680830239 | 25 | 0.881005608176 |
| 35 | 0.921875 | 1.197891286235 | 28 | 0.938498361495 |
| 36 | 0.9375 | 1.213027194718 | 32 | 0.999503608929 |
| 37 | 0.953125 | 1.229145372314 | 28 | 1.064272482884 |
| 38 | 0.96875 | 1.246306733527 | 73 | 1.133078732254 |
| 39 | 0.984375 | 1.264576661419 | 30 | 1.206221010047 |
| 40 | 1.0 | 1.284025416716 | 28 | 1.284025416716 |
| 41 | 1.015625 | 1.304728587948 | 32 | 1.366848329411 |
| 42 | 1.03125 | 1.326767588905 | 36 | 1.455079554519 |
| 43 | 1.046875 | 1.350230207899 | 33 | 1.549145843575 |
| 44 | 1.0625 | 1.375211215203 | 77 | 1.649514819408 |
| 45 | 1.078125 | 1.401813035066 | 35 | 1.756699364992 |
| 46 | 1.09375 | 1.430146491005 | 33 | 1.871262536677 |
| 47 | 1.109375 | 1.460331631535 | 38 | 1.993823068902 |
| 48 | 1.125 | 1.492498647645 | 41 | 2.125061551042 |
| the first value of $y_{44.5}$ for $x_{44.5}=1.0703125$ | | | | |
| 1.3885121251 by 2_{-1}^{+1} | | 1.3883027775 by 5_{-1}^{+4} | | |
| 1.3883028032 by 6_{-2}^{+4} | | 1.3583545959 by 7_{-3}^{+4} | | |
| 1.3883028032 by 8_{-4}^{+4} | | 1.3883028032 by 9_{-5}^{+4} | | |
| the first value of $y_{45.5}$ for $x_{45.5}=1.0859375$ | | | | |
| 1.4159797630 by 2_{-1}^{+1} | | 1.4157560432 by 5_{-2}^{+3} | | |
| 1.4157560320 by 6_{-3}^{+3} | | 1.3880657641 by 7_{-4}^{+3} | | |
| 1.4157560320 by 8_{-5}^{+3} | | 1.4157560320 by 9_{-6}^{+3} | | |
| the first value of $y_{46.5}$ for $x_{46.5}=1.1015625$ | | | | |
| 1.4452390613 by 2_{-1}^{+1} | | 1.4449997015 by 5_{-3}^{+2} | | |
| 1.4449997129 by 6_{-4}^{+2} | | 1.4449997129 by 7_{-5}^{+2} | | |
| 1.4449997129 by 8_{-6}^{+2} | | 1.4449997129 by 9_{-7}^{+2} | | |
| the first value of $y_{47.5}$ for $x_{47.5}=1.1171875$ | | | | |
| 1.4764151396 by 2_{-1}^{+1} | | 1.5062016010 by 5_{-4}^{+1} | | |
| 1.4761588448 by 6_{-5}^{+1} | | 1.4761588450 by 7_{-6}^{+1} | | |
| 1.5233256170 by 8_{-7}^{+1} | | 1.4761588448 by 9_{-8}^{+1} | | |
| 49 | 1.1328125 | 1.509369065677 | 35 | 2.194166311192 |
| 50 | 1.140625 | 1.526788892413 | 35 | 2.265727365721 |

Table 3 ($y' = x^3y$, seven nodes, 4 times) 3/6

| S=step | | E=(error of y) $\times 10^{12}$ | | |
|--------|-----------|------------------------------------|-----|-----------------|
| S | x | y | E | y' |
| 51 | 1.1484375 | 1.544777727169 | 36 | 2.339850416297 |
| 52 | 1.15625 | 1.563356015688 | 41 | 2.416646492390 |
| 53 | 1.1640625 | 1.582545092894 | 36 | 2.496232250926 |
| 54 | 1.171875 | 1.602367228142 | 44 | 2.578730294694 |
| 55 | 1.1796875 | 1.622845672891 | 38 | 2.664269510425 |
| 56 | 1.1875 | 1.644004711287 | 38 | 2.752985428398 |
| 57 | 1.1953125 | 1.665869713466 | 39 | 2.845020604490 |
| 58 | 1.203125 | 1.688467192050 | 44 | 2.940525026658 |
| 59 | 1.2109375 | 1.711824861854 | 40 | 3.039656547279 |
| 60 | 1.21875 | 1.735971703160 | 47 | 3.142581343376 |
| 61 | 1.2265625 | 1.760938028563 | 41 | 3.249474406323 |
| 62 | 1.234375 | 1.786755553941 | 41 | 3.360520063628 |
| 63 | 1.2421875 | 1.813457473474 | 42 | 3.475912534484 |
| 64 | 1.25 | 1.841078539241 | 47 | 3.595856521955 |
| 65 | 1.2578125 | 1.869655145541 | 44 | 3.720567844124 |
| 66 | 1.265625 | 1.899225418381 | 51 | 3.850274107246 |
| 67 | 1.2734375 | 1.929829310267 | 45 | 3.985215423560 |
| 68 | 1.28125 | 1.961508700951 | 45 | 4.125645177559 |
| 69 | 1.2890625 | 1.994307504222 | 47 | 4.271830843641 |
| 70 | 1.296875 | 2.028271781378 | 52 | 4.424054859385 |
| 71 | 1.3046875 | 2.063449861713 | 48 | 4.582615558283 |
| 72 | 1.3125 | 2.099892470601 | 56 | 4.747828166561 |
| 73 | 1.3203125 | 2.137652865507 | 50 | 4.920025868466 |
| 74 | 1.328125 | 2.176786980774 | 50 | 5.099560945769 |
| 75 | 1.3359375 | 2.217353581464 | 52 | 5.286805996383 |
| 76 | 1.34375 | 2.259414427141 | 57 | 5.482155238608 |
| 77 | 1.3515625 | 2.303034446155 | 54 | 5.686025907252 |
| 78 | 1.359375 | 2.348281921275 | 62 | 5.898859748861 |
| 79 | 1.3671875 | 2.395228687298 | 57 | 6.121124623294 |
| 80 | 1.375 | 2.443950341773 | 57 | 6.353316220507 |
| 81 | 1.3828125 | 2.494526469465 | 59 | 6.595959900784 |
| 82 | 1.390625 | 2.547040881797 | 64 | 6.849612668609 |
| 83 | 1.3984375 | 2.601581872238 | 62 | 7.114865290423 |
| 84 | 1.40625 | 2.658242488874 | 69 | 7.392344567828 |
| 85 | 1.4140625 | 2.717120825287 | 65 | 7.682715778189 |
| 86 | 1.421875 | 2.778320331351 | 65 | 7.986685296694 |
| 87 | 1.4296875 | 2.841950145118 | 68 | 8.305003413678 |
| 88 | 1.4375 | 2.908125447613 | 73 | 8.638467363551 |
| 89 | 1.4453125 | 2.976967842151 | 71 | 8.987924582272 |
| 90 | 1.453125 | 3.048605760089 | 79 | 9.354276212189 |
| 91 | 1.4609375 | 3.123174894899 | 75 | 9.738480874181 |
| 92 | 1.46875 | 3.200818666964 | 75 | 10.141558729866 |
| 93 | 1.4765625 | 3.281688721196 | 79 | 10.564595857172 |
| 94 | 1.484375 | 3.365945460205 | 84 | 11.008748965999 |
| 95 | 1.4921875 | 3.453758615723 | 83 | 11.475250482319 |
| 96 | 1.5 | 3.545307861314 | 91 | 11.965414031936 |
| 97 | 1.5078125 | 3.640783469507 | 88 | 12.480640357466 |
| 98 | 1.515625 | 3.740387017097 | 88 | 13.022423706265 |
| 99 | 1.5234375 | 3.844332142211 | 93 | 13.59258728876 |
| 100 | 1.53125 | 3.952845357449 | 98 | 14.192147932696 |

Table 3 ($y' = x^3y$, seven nodes, 4 times) 4/6

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{12}$ | | |
|---|------------|---|-------------------------------|-----------------|
| S | x | y | E | y' |
| 101 | 1.5390625 | 4.066166923584 | 98 | 14.823609738856 |
| 102 | 1.546875 | 4.184551788748 | 107 | 15.488687195095 |
| 103 | 1.5546875 | 4.308270598365 | 105 | 16.189457401851 |
| 104 | 1.5625 | 4.437610781826 | 105 | 16.928141715340 |
| the first value of $y_{100.5}$ for $x_{100.5}=1.53515625$ | | | | |
| | | 4.0095061405 by 2_{-1}^{+1} | 4.0088894879 by 5_{-1}^{+4} | |
| | | 4.0088895176 by 6_{-2}^{+4} | 4.0088895181 by 7_{-3}^{+4} | |
| | | 4.0088895181 by 8_{-4}^{+4} | 4.0088895181 by 9_{-5}^{+4} | |
| the first value of $y_{101.5}$ for $x_{101.5}=1.54296875$ | | | | |
| | | 4.1253593562 by 2_{-1}^{+1} | 4.1247099218 by 5_{-2}^{+3} | |
| | | 4.1247099091 by 6_{-3}^{+3} | 4.1247099087 by 7_{-4}^{+3} | |
| | | 4.1247099087 by 8_{-5}^{+3} | 4.1247099087 by 9_{-6}^{+3} | |
| the first value of $y_{102.5}$ for $x_{102.5}=1.55078125$ | | | | |
| | | 4.2464111936 by 2_{-1}^{+1} | 4.2457268795 by 5_{-3}^{+2} | |
| | | 4.2457268923 by 6_{-2}^{+2} | 4.2457268928 by 7_{-5}^{+2} | |
| | | 4.2457268928 by 8_{-6}^{+2} | 4.2457268928 by 9_{-7}^{+2} | |
| the first value of $y_{103.5}$ for $x_{103.5}=1.55859375$ | | | | |
| | | 4.3729406901 by 2_{-1}^{+1} | 4.4068882326 by 5_{-4}^{+1} | |
| | | 4.3722193670 by 6_{-5}^{+1} | 4.3722193670 by 7_{-6}^{+1} | |
| | | 4.3722193670 by 8_{-7}^{+1} | 4.3722193670 by 9_{-8}^{+1} | |
| 105 | 1.56640625 | 4.504483585810 | 114 | 17.312437981238 |
| 106 | 1.5703125 | 4.572877722035 | 116 | 17.707116795748 |
| 107 | 1.57421875 | 4.642834375212 | 118 | 18.112501471139 |
| 108 | 1.578125 | 4.714396014826 | 115 | 18.528926576505 |
| 109 | 1.58203125 | 4.787606439978 | 122 | 18.956738368188 |
| 110 | 1.5859375 | 4.862510825848 | 116 | 19.396295237704 |
| 111 | 1.58984375 | 4.939155772090 | 125 | 19.847968178856 |
| 112 | 1.59375 | 5.017589352905 | 128 | 20.312141273566 |
| 113 | 1.59765625 | 5.097861169227 | 129 | 20.789212198735 |
| 114 | 1.6015625 | 5.180022402813 | 127 | 21.279592753904 |
| 115 | 1.60546875 | 5.264125872505 | 133 | 21.783709411345 |
| 116 | 1.609375 | 5.305226092603 | 128 | 22.302003889051 |
| 117 | 1.61328125 | 5.438379333660 | 137 | 22.834933748517 |
| 118 | 1.6171875 | 5.528643685488 | 141 | 23.382973017144 |
| 119 | 1.62109375 | 5.621079122803 | 142 | 23.946612837839 |
| 120 | 1.625 | 5.715747573371 | 140 | 24.526362145889 |
| 121 | 1.62890625 | 5.812712988891 | 147 | 25.122748375050 |
| 122 | 1.6328125 | 5.912041418645 | 142 | 25.736318193645 |
| 123 | 1.63671875 | 6.013801086195 | 152 | 26.367638272926 |
| 124 | 1.640625 | 6.118062468982 | 155 | 27.017296087855 |
| 125 | 1.64453125 | 6.224898381282 | 157 | 27.685900753301 |
| 126 | 1.6484375 | 6.334384060404 | 156 | 28.374083896129 |
| 127 | 1.65234375 | 6.446597256401 | 163 | 29.082500565501 |
| 128 | 1.65625 | 6.561618325370 | 159 | 29.811930182679 |
| 129 | 1.66015625 | 6.679530326648 | 168 | 30.562777533029 |
| 130 | 1.6640625 | 6.800419123815 | 173 | 31.336073800869 |

Table 3 ($y' = x^3y$, seven nodes, 4 times) 5/6

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{12}$ | | |
|-----------------|------------|---|-----|------------------|
| S | x | y | E | y |
| 131 | 1.66796875 | 6.924373489973 | 174 | 32.132477650687 |
| 132 | 1.671875 | 7.051485217256 | 174 | 32.952776355753 |
| 133 | 1.67578125 | 7.181849230881 | 181 | 33.797786976988 |
| 134 | 1.6796875 | 7.315563707853 | 177 | 34.668357594031 |
| 135 | 1.68359375 | 7.452730200703 | 187 | 35.565368591802 |
| 136 | 1.6875 | 7.593453766191 | 193 | 36.489734003891 |
| 137 | 1.69140625 | 7.737843099540 | 194 | 37.442402916982 |
| 138 | 1.6953125 | 7.886010674183 | 194 | 38.424360938139 |
| 139 | 1.69921875 | 8.038072887400 | 202 | 39.436631728499 |
| 140 | 1.703125 | 8.194150212046 | 199 | 40.480278606244 |
| 141 | 1.70703125 | 8.354367354771 | 210 | 41.556406222945 |
| 142 | 1.7109375 | 8.518853420800 | 216 | 42.666162315549 |
| 143 | 1.71484375 | 8.687742085843 | 218 | 43.810739539193 |
| 144 | 1.71875 | 8.861171775258 | 219 | 44.991377383685 |
| 145 | 1.72265625 | 9.039285850888 | 227 | 46.209364178233 |
| 146 | 1.7265625 | 9.22232805873 | 225 | 47.466039188497 |
| 147 | 1.73046875 | 9.410166467940 | 236 | 48.762794811184 |
| 148 | 1.734375 | 9.603246211319 | 243 | 50.101078869751 |
| 149 | 1.73828125 | 9.801637177962 | 246 | 51.482397017709 |
| 150 | 1.7421875 | 10.005510508313 | 248 | 52.908315253800 |
| 151 | 1.74609375 | 10.215043582126 | 256 | 54.380462555059 |
| 152 | 1.75 | 10.430420269795 | 255 | 55.900533633433 |
| 153 | 1.75390625 | 10.651831194787 | 267 | 57.470291822816 |
| 154 | 1.7578125 | 10.879474007477 | 275 | 59.091572101793 |
| 155 | 1.76171875 | 11.113553671202 | 278 | 60.766284260424 |
| 156 | 1.765625 | 11.354282760911 | 281 | 62.496416217308 |
| 157 | 1.76953125 | 11.601881775088 | 291 | 64.284037494917 |
| 158 | 1.7734375 | 11.856579461546 | 291 | 66.131302861119 |
| 159 | 1.77734375 | 12.118613157879 | 303 | 68.040456145971 |
| 160 | 1.78125 | 12.388229147030 | 313 | 70.013834241517 |
| 161 | 1.78515625 | 12.665683028998 | 317 | 72.053871295448 |
| 162 | 1.7890625 | 12.951240109251 | 321 | 74.163103107622 |
| 163 | 1.79296875 | 13.245175804733 | 332 | 76.344171740180 |
| 164 | 1.796875 | 13.547776068297 | 333 | 78.599830352292 |
| 165 | 1.80078125 | 13.859337832561 | 347 | 80.932948271743 |
| 166 | 1.8046875 | 14.180169473929 | 358 | 83.346516314467 |
| 167 | 1.80859375 | 14.510591298012 | 363 | 85.843652366467 |
| 168 | 1.8125 | 14.850936047344 | 369 | 88.427607240886 |
| 169 | 1.81640625 | 15.201549432525 | 381 | 91.101770824921 |
| 170 | 1.8203125 | 15.562790687962 | 384 | 93.869678531883 |
| 171 | 1.82421875 | 15.935033153508 | 398 | 96.735018075047 |
| 172 | 1.828125 | 16.318664883088 | 411 | 99.761636579122 |
| 173 | 1.83203125 | 16.714089281916 | 418 | 102.773548048797 |
| 174 | 1.8359375 | 17.121725773564 | 426 | 105.954941212457 |
| 175 | 1.83984375 | 17.542010498420 | 439 | 109.250187761282 |
| 176 | 1.84375 | 17.975397045142 | 444 | 112.663851005070 |
| 177 | 1.84765625 | 18.422357216834 | 460 | 116.200694967677 |
| 178 | 1.8515625 | 18.883381833534 | 475 | 119.865693944567 |
| 179 | 1.85546875 | 19.358981573085 | 484 | 123.664042254901 |
| 180 | 1.859375 | 19.849687852212 | 494 | 127.601166272528 |

Table 3 ($y' = x^3y$, seven nodes, 4 times) 6/6

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{12}$ | | |
|-----------------|------------|---|------|------------------|
| S | x | y | E | y' |
| 181 | 1.86328125 | 20.356053749875 | 509 | 131.682732587669 |
| 182 | 1.8671875 | 20.878654975105 | 517 | 135.914662622968 |
| 183 | 1.87109375 | 21.418090881683 | 535 | 140.303143441822 |
| 184 | 1.875 | 21.974985531888 | 553 | 144.854640957270 |
| 185 | 1.87890625 | 22.549988812111 | 563 | 149.575913519248 |
| 186 | 1.8828125 | 23.143777602874 | 576 | 154.474026210578 |
| 187 | 1.88671875 | 23.757057006125 | 594 | 159.556365891015 |
| 188 | 1.890625 | 24.390561632820 | 604 | 164.830657031248 |
| 189 | 1.89453125 | 25.045056954049 | 626 | 170.304978381270 |
| 190 | 1.8984375 | 25.721340718855 | 646 | 175.987780518612 |
| 191 | 1.90234375 | 26.420244442497 | 660 | 181.887904327331 |
| 192 | 1.90625 | 27.142634968783 | 676 | 188.014600459272 |
| 193 | 1.91015625 | 27.889416110389 | 698 | 194.377549833007 |
| 194 | 1.9140625 | 28.661530371391 | 711 | 200.986885229577 |
| 195 | 1.91796875 | 29.459960756461 | 736 | 207.853214047676 |
| 196 | 1.921875 | 30.285732671200 | 761 | 214.987642283091 |
| 197 | 1.92578125 | 31.139915918752 | 778 | 222.401799803936 |
| 198 | 1.9296875 | 32.023626797774 | 798 | 230.107866995063 |
| 199 | 1.93359375 | 32.938030307251 | 824 | 238.118602850253 |
| 200 | 1.9375 | 33.884342464025 | 842 | 246.447374596135 |
| 201 | 1.94140625 | 34.863832739235 | 871 | 255.108188936684 |
| 202 | 1.9453125 | 35.877826620008 | 900 | 264.115725011117 |
| 203 | 1.94921875 | 36.927708303504 | 922 | 273.485369166440 |
| 204 | 1.953125 | 38.014923530481 | 948 | 283.233251649744 |
| 205 | 1.95703125 | 39.140982566079 | 979 | 293.376285332400 |
| 206 | 1.9609375 | 40.307463336059 | 1002 | 303.932206586076 |
| 207 | 1.96484375 | 41.516014727168 | 1037 | 314.919618437490 |

Table 4 ($y' = x^3y$, seven nodes, 5 times) 1/6 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

| S | x | y | E | y' |
|-----|---------|----------------|------|-----------------|
| 1 | -0.1875 | 1.000309038221 | 0 | -0.006593833992 |
| 2 | -0.125 | 1.000061037019 | 0 | -0.001953244213 |
| 3 | -0.0625 | 1.000003814705 | 0 | -0.000244141556 |
| 4 | 0 | 1.000000000000 | 0 | 0.000000000000 |
| 5 | 0.0625 | 1.000003814705 | 0 | 0.000244141556 |
| 6 | 0.125 | 1.000061037019 | 0 | 0.001953244213 |
| 7 | 0.1875 | 1.000309038221 | 0 | 0.006593833992 |
| 8 | 0.25 | 1.000977039504 | 12 | 0.015640266242 |
| 9 | 0.3125 | 1.002387030274 | 52 | 0.030590424508 |
| 10 | 0.375 | 1.004956088785 | 150 | 0.052995731245 |
| 11 | 0.4375 | 1.009201161273 | 339 | 0.084510741777 |
| 12 | 0.5 | 1.015747709245 | 659 | 0.126968463656 |
| 13 | 0.5625 | 1.025344065474 | 1166 | 0.182489214778 |
| 14 | 0.625 | 1.038883911151 | 1965 | 0.253633767371 |

the first approximate value of $y_{10.5}$ for $x_{10.5}=0.40625$ 1.0070762503 by 2_{-1}^{+1} 1.0068320337 by 5_{-1}^{+4} 1.0068326483 by 6_{-2}^{+4} 1.0068327038 by 7_{-3}^{+4} 1.0068327097 by 8_{-4}^{+4} 1.0068327103 by 9_{-5}^{+4} the first approximate value of $y_{11.5}$ for $x_{11.5}=0.46875$ 1.0124744353 by 2_{-1}^{+1} 1.0121433866 by 5_{-2}^{+3} 1.0121431232 by 6_{-3}^{+3} 1.0121430836 by 7_{-4}^{+3} 1.0121430777 by 8_{-5}^{+3} 1.0121430769 by 9_{-6}^{+3} the first approximate value of $y_{12.5}$ for $x_{12.5}=0.53125$ 1.0205458874 by 2_{-1}^{+1} 1.0201122129 by 5_{-3}^{+2} 1.0201124763 by 6_{-4}^{+2} 1.0201125318 by 7_{-5}^{+2} 1.0201125442 by 8_{-6}^{+2} 1.0201125441 by 9_{-7}^{+2} the first approximate value of $y_{13.5}$ for $x_{13.5}=0.59375$ 1.0321139883 by 2_{-1}^{+1} 1.0396757999 by 5_{-4}^{+1} 1.0315589048 by 6_{-5}^{+1} 1.0315587382 by 7_{-6}^{+1} 1.0315586994 by 8_{-7}^{+1} 1.0315586919 by 9_{-8}^{+1}

| | | | | |
|----|---------|----------------|------|----------------|
| 15 | 0.65625 | 1.047459684126 | 716 | 0.296036503134 |
| 16 | 0.6875 | 1.057440095792 | 701 | 0.343616398315 |
| 17 | 0.71875 | 1.068995362947 | 739 | 0.396925860015 |
| 18 | 0.75 | 1.082314240257 | 1228 | 0.456601320108 |
| 19 | 0.78125 | 1.097606893184 | 814 | 0.523379751770 |
| 20 | 0.8125 | 1.115108344838 | 2058 | 0.598118416408 |
| 21 | 0.84375 | 1.135082583244 | 859 | 0.681818557312 |
| 22 | 0.875 | 1.157827507580 | 2058 | 0.775653974804 |
| 23 | 0.90625 | 1.183680831147 | 933 | 0.881005608851 |
| 24 | 0.9375 | 1.213027196101 | 1416 | 0.999503610069 |
| 25 | 0.96875 | 1.246306734531 | 1077 | 1.133078733167 |

the first value of $y_{21.5}$ for $x_{21.5}=0.859375$ 1.1464550454 by 2_{-1}^{+1} 1.1460883681 by 5_{-1}^{+4} 1.1460886022 by 6_{-2}^{+4} 1.1460886120 by 7_{-3}^{+4} 1.1460886126 by 8_{-4}^{+4} 1.1460886126 by 9_{-5}^{+4} the first value of $y_{22.5}$ for $x_{22.5}=0.890625$ 1.1707541694 by 2_{-1}^{+1} 1.1703428799 by 5_{-2}^{+3} 1.1703427775 by 6_{-3}^{+3} 1.1703427725 by 7_{-4}^{+3} 1.1703427719 by 8_{-5}^{+3} 1.1703427719 by 9_{-6}^{+3} the first value of $y_{123.5}$ for $x_{123.5}=0.921875$ Table 4 ($y' = x^3y$, seven nodes, 5 times) 2/6 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

| | |
|---|-------------------------------|
| 1.1983540136 by 2_{-1}^{+1} | 1.1978911757 by 5_{-3}^{+2} |
| 1.1978912761 by 6_{-4}^{+2} | 1.1978912860 by 7_{-5}^{+2} |
| 1.1978912870 by 8_{-6}^{+2} | 1.1978912869 by 9_{-7}^{+2} |
| the first value of $y_{24.5}$ for $x_{24.5}=0.953125$ | |
| 1.2296669653 by 2_{-1}^{+1} | 1.2838824142 by 5_{-4}^{+1} |
| 1.2291454087 by 6_{-5}^{+1} | 1.2291453790 by 7_{-6}^{+1} |
| 1.2291453753 by 8_{-7}^{+1} | 1.2291453755 by 9_{-8}^{+1} |

| S | x | y | E | y' |
|-----|----------|----------------|------|-----------------|
| 26 | 0.984375 | 1.264576662399 | 1011 | 1.206221010982 |
| 27 | 1.0 | 1.284025417702 | 1014 | 1.284025417702 |
| 28 | 1.015625 | 1.304728588957 | 1040 | 1.366848330468 |
| 29 | 1.03125 | 1.326767590380 | 1511 | 1.455079561366 |
| 30 | 1.046875 | 1.350230208957 | 1091 | 1.549145844789 |
| 31 | 1.0.625 | 1.375211216310 | 1184 | 1.649514207346 |
| 32 | 1.078125 | 1.401813036169 | 1138 | 1.756699366374 |
| 33 | 1.09375 | 1.430146492106 | 1134 | 1.871262538118 |
| 34 | 1.109375 | 1.460331632678 | 1181 | 1.993823070462 |
| 35 | 1.125 | 1.492498649256 | 1652 | 2.125061553335 |
| 36 | 1.140625 | 1.526788893629 | 1252 | 2.265727367527 |
| 37 | 1.15625 | 1.563356016988 | 1341 | 2.416646494400 |
| 38 | 1.171875 | 1.602367229422 | 1324 | 2.578730296754 |
| 39 | 1.1875 | 1.644004712563 | 1313 | 2.752958430534 |
| 40 | 1.203125 | 1.688467193393 | 1386 | 2.940525289958 |
| 41 | 1.21875 | 1.735971704974 | 1861 | 3.142581346660 |
| 42 | 1.234375 | 1.786755555388 | 1488 | 3.360520066349 |
| 43 | 1.25 | 1.841078540771 | 1577 | 3.595856524943 |
| 44 | 1.265625 | 1.899225429929 | 1598 | 3.850274110383 |
| 45 | 1.28125 | 1.961508720498 | 1584 | 4.125645180796 |
| 46 | 1.296875 | 2.027271783014 | 1693 | 4.424054862966 |
| 47 | 1.3125 | 2.099892472725 | 2180 | 4.747828171365 |
| 48 | 1.328125 | 2.176786982568 | 1844 | 5.099560949971 |
| 49 | 1.34375 | 2.259414429058 | 1942 | 5.482155243181 |
| 50 | 1.359375 | 2.348281923229 | 2015 | 5.0898859753769 |
| 51 | 1.375 | 2.443950343725 | 2008 | 6.353316225581 |
| 52 | 1.390625 | 2.547040883903 | 2169 | 6.849612674270 |

the first value of $y_{48.5}$ for $x_{48.5}=1.3359375$ 2.2181007058 by 2_{-1}^{+1} 2.2173533987 by 5_{-1}^{+4} 2.2173535777 by 6_{-2}^{+4} 2.2173535830 by 7_{-3}^{+4} 2.2173535832 by 8_{-4}^{+4} 2.2173535830 by 9_{-5}^{+4} the first value of $y_{49.5}$ for $x_{49.5}=1.3515625$ 2.3038481761 by 2_{-1}^{+1} 2.3030345289 by 5_{-2}^{+3} 2.3030344522 by 6_{-3}^{+3} 2.3030344084 by 7_{-4}^{+3} 2.3030344481 by 8_{-5}^{+3} 2.3030344452 by 9_{-6}^{+3} the first value of $y_{50.5}$ for $x_{50.5}=1.359375$ 2.3961161335 by 2_{-1}^{+1} 2.3952286067 by 5_{-3}^{+2} 2.3952286835 by 6_{-4}^{+2} 2.3952286888 by 7_{-5}^{+2} 2.3952286891 by 8_{-6}^{+2} 2.3952286896 by 9_{-7}^{+2} the first value of $y_{51.5}$ for $x_{51.5}=1.390625$ 2.4954956138 by 2_{-1}^{+1} 2.5144254251 by 5_{-4}^{+1} 2.4945264892 by 6_{-5}^{+1} 2.4945264733 by 7_{-6}^{+1} 2.4945264722 by 8_{-7}^{+1} 2.4945264742 by 9_{-8}^{+1}

Table 4 ($y' = x^3y$, seven nodes, 5 times) 3/6 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

| S | x | y | E | y' |
|-----|-----------|-----------------|-------|-----------------|
| 53 | 1.3987375 | 2.061581874406 | 2229 | 7.114865296350 |
| 54 | 1.40625 | 2.658242491085 | 2281 | 7.392344573979 |
| 55 | 1.4140625 | 2.717120827551 | 2329 | 7.682715784590 |
| 56 | 1.421875 | 2.778320333580 | 2294 | 7.986685303103 |
| 57 | 1.4296875 | 2.841950147483 | 2436 | 8.305003205912 |
| 58 | 1.4375 | 2.908125450018 | 2478 | 8.638467370696 |
| 59 | 1.4453125 | 2.976967844627 | 2547 | 8.987924589462 |
| 60 | 1.453125 | 3.048605762626 | 2615 | 9.354276219972 |
| 61 | 1.4609375 | 3.123174897497 | 2673 | 9.738480882282 |
| 62 | 1.46875 | 3.200818669544 | 2654 | 10.141558738039 |
| 63 | 1.4765625 | 3.281688723924 | 2807 | 10.564595865953 |
| 64 | 1.484375 | 3.365945462990 | 2870 | 11.008748975110 |
| 65 | 1.4921875 | 3.453758618591 | 2950 | 11.475250491847 |
| 66 | 1.5 | 3.545307864263 | 3039 | 11.965414041888 |
| 67 | 1.5078125 | 3.640783472532 | 3113 | 12.480640367836 |
| 68 | 1.515625 | 3.740387020124 | 3115 | 13.022423716802 |
| 69 | 1.5234375 | 3.844332145403 | 3284 | 13.592358740161 |
| 70 | 1.53125 | 3.952845360722 | 3371 | 14.192147944445 |
| 71 | 1.5390625 | 4.066166926955 | 3470 | 14.823609751147 |
| 72 | 1.546875 | 4.184551792227 | 3585 | 15.488687207971 |
| 73 | 1.5546875 | 4.308270601901 | 3680 | 16.189457415287 |
| 74 | 1.5625 | 4.437610785430 | 3709 | 16.928141729087 |
| 75 | 1.5703125 | 4.572877725823 | 3904 | 17.707116810414 |
| 76 | 1.578125 | 4.714396018732 | 4021 | 18.528926591857 |
| 77 | 1.5859375 | 4.862510829878 | 4146 | 19.396295253779 |
| 78 | 1.59375 | 5.017589357074 | 4297 | 20.312141290443 |
| 79 | 1.6015625 | 5.180022407108 | 4421 | 21.279592771547 |
| 80 | 1.609375 | 5.350226096961 | 4487 | 22.302003907219 |
| 81 | 1.6171875 | 5.528643690064 | 4717 | 23.382973036499 |
| 82 | 1.625 | 5.715747578106 | 4875 | 24.526362166208 |
| 83 | 1.6328125 | 5.912041423541 | 5037 | 25.736318214956 |
| 84 | 1.640625 | 6.118062474062 | 5240 | 27.017296110291 |
| 85 | 1.6484375 | 6.334384065653 | 5404 | 28.374083919639 |
| 86 | 1.65625 | 6.561618330730 | 5519 | 29.811830207036 |
| 87 | 1.6640625 | 6.800419129442 | 5800 | 31.336073826798 |
| 88 | 1.671875 | 7.051485223097 | 6015 | 32.952776383053 |
| 89 | 1.6796875 | 7.315563713907 | 6231 | 34.668357622721 |
| 90 | 1.6875 | 7.593453772494 | 6495 | 36.489734034178 |
| 91 | 1.6953125 | 7.886010680714 | 6726 | 38.424360969963 |
| 92 | 1.703125 | 8.194150218758 | 6911 | 40.480278639403 |
| 93 | 1.7109375 | 8.518853427848 | 7264 | 42.666350849002 |
| 94 | 1.71875 | 8.861171782599 | 7560 | 44.991377420958 |
| 95 | 1.7265625 | 9.22232813503 | 7855 | 47.466029227769 |
| 96 | 1.734375 | 9.603246219287 | 8211 | 50.101078113222 |
| 97 | 1.7421875 | 10.005510516599 | 8533 | 52.908315297616 |
| 98 | 1.75 | 10.430420278359 | 8196 | 55.900533679332 |
| 99 | 1.7578125 | 10.879474016480 | 9278 | 59.091572150692 |
| 100 | 1.765625 | 11.354282770319 | 9690 | 62.496416269093 |
| 101 | 1.7734375 | 11.856579471357 | 10102 | 66.131302915841 |
| 102 | 1.78125 | 12.388229157309 | 10591 | 70.013834299606 |
| 103 | 1.7890625 | 12.951240119979 | 11049 | 74.163103169052 |
| 104 | 1.796875 | 13.547776079446 | 11483 | 78.599830416976 |
| 105 | 1.8046875 | 14.180169485670 | 12098 | 83.346516383472 |

Table 4 ($y' = x^3y$, seven nodes, 5 times) 4/6 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

| | | | | |
|---|------------------|-----------------|------------------|------------------|
| | | | | |
| the first value of $y_{101.5}$ for $x_{101.5}=1.77734375$ | | | | |
| 12.1224043143 | by 2_{-1}^{+1} | 12.1186127962 | by 5_{-1}^{+4} | |
| 12.1186131600 | by 6_{-2}^{+4} | 12.1186131677 | by 7_{-3}^{+4} | |
| 12.1186131679 | by 8_{-4}^{+4} | 12.1186131678 | by 9_{-5}^{+4} | |
| the first value of $y_{102.5}$ for $x_{102.5}=1.78515625$ | | | | |
| 12.6697346386 | by 2_{-1}^{+1} | 12.6656832012 | by 5_{-2}^{+3} | |
| 12.6656830453 | by 6_{-3}^{+3} | 12.6656830398 | by 7_{-4}^{+3} | |
| 12.6656830395 | by 8_{-5}^{+3} | 12.6656830395 | by 9_{-6}^{+3} | |
| the first value of $y_{103.5}$ for $x_{103.5}=1.79296875$ | | | | |
| 13.2495080997 | by 2_{-1}^{+1} | 13.2451756516 | by 5_{-2}^{+3} | |
| 13.2451758075 | by 6_{-2}^{+2} | 13.2451758152 | by 7_{-3}^{+2} | |
| 13.2451758156 | by 8_{-4}^{+2} | 13.2451758157 | by 9_{-5}^{+2} | |
| the first value of $y_{104.5}$ for $x_{104.5}=1.80078125$ | | | | |
| 13.8639727826 | by 2_{-1}^{+1} | 13.9701208067 | by 5_{-1}^{+4} | |
| 13.8593378689 | by 6_{-5}^{+1} | 13.8593378457 | by 7_{-6}^{+1} | |
| 13.8593378441 | by 8_{-7}^{+1} | 13.8593378440 | by 9_{-8}^{+1} | |
| 106 | 1.80859375 | 14.510591310025 | 12376 | 85.843652437537 |
| 107 | 1.8125 | 14.850936059644 | 12669 | 88.427607314126 |
| 108 | 1.81640625 | 15.201549445111 | 12966 | 91.101770900345 |
| 109 | 1.8203125 | 15.562790700779 | 13200 | 93.869678609189 |
| 110 | 1.82421875 | 15.935033166699 | 13590 | 96.735018155127 |
| 111 | 1.828125 | 16.318664896598 | 13921 | 99.701636661665 |
| 112 | 1.83203125 | 16.714089295751 | 14252 | 102.773548133865 |
| 113 | 1.8359375 | 17.121725787744 | 14605 | 105.954941300208 |
| 114 | 1.83984375 | 17.542010512941 | 14960 | 109.250187851716 |
| 115 | 1.84375 | 17.975397059954 | 15257 | 112.663851097912 |
| 116 | 1.84765625 | 18.422357232082 | 15709 | 116.200695063856 |
| 117 | 1.8515625 | 18.883381849167 | 16108 | 119.865694043801 |
| 118 | 1.85546875 | 19.358981589106 | 16505 | 123.664042651354 |
| 119 | 1.859375 | 19.849687868650 | 16931 | 127.601166378196 |
| 120 | 1.86328125 | 20.356053766723 | 17357 | 131.682732696659 |
| 121 | 1.8671875 | 20.878654992320 | 17731 | 135.914662735032 |
| 122 | 1.87109375 | 21.418090899408 | 18261 | 140.303143557937 |
| 123 | 1.875 | 21.974985550079 | 18744 | 144.854641077184 |
| 124 | 1.87890625 | 22.549988830770 | 19223 | 149.575913643017 |
| 125 | 1.8828125 | 23.143777622038 | 19740 | 154.474026338486 |
| 126 | 1.88671875 | 23.757057025786 | 20255 | 159.556366023061 |
| 127 | 1.890625 | 24.390561652940 | 20724 | 164.830657167220 |
| 128 | 1.89453125 | 25.045056974775 | 21351 | 170.304978522201 |
| 129 | 1.8984375 | 25.721340740147 | 21938 | 175.987780664290 |
| 130 | 1.90234375 | 26.420244646357 | 22519 | 181.887904477821 |
| 131 | 1.90625 | 27.142634991256 | 23149 | 188.014600614938 |
| 132 | 1.91015625 | 27.889416133468 | 23776 | 194.377549993856 |
| 133 | 1.9140625 | 28.661530395045 | 24364 | 200.986885395445 |
| 134 | 1.91796875 | 29.459960780838 | 25113 | 207.853214219665 |
| 135 | 1.921875 | 30.285732696269 | 25829 | 214.987642461042 |
| 136 | 1.92578125 | 31.139915944514 | 26540 | 222.401799987931 |
| 137 | 1.9296875 | 32.023626824285 | 27309 | 230.107867185561 |
| 138 | 1.93359375 | 32.938030334506 | 28078 | 238.118603047284 |
| 139 | 1.9375 | 33.884342491999 | 28815 | 246.447374799594 |
| 140 | 1.94140625 | 34.863832768081 | 29717 | 255.108189147762 |
| 141 | 1.9453125 | 35.877826649703 | 30595 | 264.115725229717 |
| 142 | 1.94921875 | 36.927708334053 | 31471 | 273.485369392684 |

Table 4 ($y' = x^3y$, seven nodes, 5 times) 5/6 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

| S | x | y | E | y |
|-----|------------|------------------|--------|-------------------|
| 143 | 1.953125 | 38.014923561949 | 32416 | 283.233251884200 |
| 144 | 1.95503125 | 39.140982598465 | 33364 | 293.376285575140 |
| 145 | 1.9609375 | 40.307463369346 | 34289 | 303.932206837024 |
| 146 | 1.96484375 | 41.516014761517 | 35386 | 314.919618698044 |
| 147 | 1.96875 | 42.768360096035 | 36469 | 326.358036405437 |
| 148 | 1.97265625 | 44.066300785181 | 37553 | 338.267935553373 |
| 149 | 1.9765625 | 45.411720182039 | 38721 | 350.670802914822 |
| 150 | 1.98046875 | 46.806587634114 | 39897 | 363.589190137032 |
| 151 | 1.984375 | 48.252962753023 | 41061 | 377.046770488458 |
| 152 | 1.98828125 | 49.752999912018 | 42405 | 391.068398823414 |
| 153 | 1.9921875 | 51.308952983609 | 43748 | 405.680174960415 |
| 154 | 1.99609375 | 52.923180332947 | 45100 | 420.909510692875 |
| 155 | 2.0 | 54.598150079695 | 46551 | 436.785200637558 |
| 156 | 2.00390625 | 56.336445644396 | 48018 | 453.337497158308 |
| 157 | 2.0078125 | 58.140771596384 | 49487 | 470.598189618386 |
| 158 | 2.01171875 | 60.013959820032 | 51150 | 488.600688221040 |
| 159 | 2.015625 | 61.958976017204 | 52826 | 507.380112714366 |
| 160 | 2.01963125 | 63.978926567354 | 54521 | 526.973386274945 |
| 161 | 2.0234375 | 66.077065764312 | 56334 | 547.419334874524 |
| 162 | 2.02734375 | 68.256803452294 | 58177 | 568.758792472850 |
| 163 | 2.03125 | 70.521713085349 | 60038 | 591.034712404293 |
| 164 | 2.03515625 | 72.875540234742 | 62111 | 614.292285337774 |
| 165 | 2.0390625 | 75.322211568309 | 64215 | 638.579064215606 |
| 166 | 2.04296875 | 77.865844337364 | 66355 | 663.945096623888 |
| 167 | 2.046875 | 80.510756389933 | 68636 | 690.443065045931 |
| 168 | 2.05078125 | 83.261476751839 | 70965 | 718.128435498828 |
| 169 | 2.0546875 | 86.122756804649 | 73334 | 747.059615090215 |
| 170 | 2.05859375 | 89.099582097434 | 75938 | 777.298119053608 |
| 171 | 2.0625 | 92.197184830051 | 78598 | 808.908747860727 |
| 172 | 2.06640625 | 95.421057495496 | 81315 | 841.959775069343 |
| 173 | 2.0703125 | 98.776964602589 | 84205 | 876.523146579044 |
| 174 | 2.07421875 | 102.270961888785 | 87167 | 912.674692022845 |
| 175 | 2.078125 | 105.909407465734 | 90198 | 950.494349105186 |
| 176 | 2.08203125 | 109.698980556615 | 93494 | 990.066401681695 |
| 177 | 2.0859375 | 113.646698515497 | 96881 | 1031.479732492361 |
| 178 | 2.08984375 | 117.759935311152 | 100353 | 1074.868091512468 |
| 179 | 2.09375 | 122.046441090627 | 104039 | 1120.210380912481 |
| 180 | 2.09765625 | 126.514362889839 | 107830 | 1167.730957725085 |
| 181 | 2.1015625 | 131.172266563651 | 111729 | 1217.499955386411 |
| 182 | 2.10546875 | 136.029160010553 | 115936 | 1269.633625384240 |
| 183 | 2.109375 | 141.094517772666 | 120276 | 1324.254700336163 |
| 184 | 2.11328125 | 146.378307099322 | 124744 | 1381.492779932855 |
| 185 | 2.1171875 | 151.891015564726 | 129477 | 1441.484741248196 |
| 186 | 2.12109375 | 157.643680338262 | 134363 | 1504.375175041111 |
| 187 | 2.125 | 163.647919213452 | 139409 | 1570.316849796272 |
| 188 | 2.12890625 | 169.915963506440 | 144820 | 1639.471205352108 |
| 189 | 2.1328125 | 176.460692943059 | 150422 | 1712.008878101555 |
| 190 | 2.13671875 | 183.295672663730 | 156210 | 1788.110259911835 |
| 191 | 2.140625 | 190.425192480578 | 162332 | 1867.966093027164 |
| 192 | 2.14453125 | 197.894308532224 | 168672 | 1951.778103397520 |
| 193 | 2.1484375 | 205.688887492625 | 175244 | 2039.759675060839 |
| 194 | 2.15234375 | 213.835653498583 | 182257 | 2132.136568369907 |
| 195 | 2.15625 | 222.352237972666 | 189541 | 2229.147685063556 |

Table 4 ($y' = x^3y$, seven nodes, 5 times) 6/6 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

| S | x | y | E | y' |
|--|---------------------------------|---------------------------------|--------|-------------------|
| 196 | 2.16015625 | 231.257232532547 | 197092 | 2331.045883418369 |
| 197 | 2.1640625 | 240.570245187326 | 205069 | 2438.098846914765 |
| 198 | 2.16796875 | 250.311960037147 | 213355 | 2550.590010118213 |
| 199 | 2.171875 | 260.504200708589 | 221970 | 2668.819545756535 |
| 200 | 2.17578125 | 271.169997771836 | 231132 | 2793.105417237506 |
| the first value of $y_{196.5}$ for $x_{196.5}=2.162109375$ | | | | |
| | 235.9137388599 by 2^{+1}_{-1} | 235.8614679160 by 5^{+4}_{-1} | | |
| | 235.8614697594 by 6^{+4}_{-2} | 235.8614677866 by 7^{+4}_{-3} | | |
| | 235.8614697872 by 8^{+4}_{-4} | 235.8614697873 by 9^{+4}_{-5} | | |
| the first value of $y_{197.5}$ for $x_{197.5}=2.166015625$ | | | | |
| | 245.4411026122 by 2^{+1}_{-1} | 245.3861791489 by 5^{+3}_{-2} | | |
| | 245.3861783589 by 6^{+3}_{-3} | 245.3861783394 by 7^{+3}_{-4} | | |
| | 245.3861783388 by 8^{+3}_{-5} | 245.3861783389 by 9^{+3}_{-6} | | |
| the first value of $y_{198.5}$ for $x_{198.5}=2.169921875$ | | | | |
| | 255.4080803729 by 2^{+1}_{-1} | 255.3503535223 by 5^{+2}_{-3} | | |
| | 255.3503543123 by 6^{+2}_{-4} | 255.3503543395 by 7^{+2}_{-5} | | |
| | 255.3503543406 by 8^{+2}_{-6} | 255.3503543407 by 9^{+2}_{-7} | | |
| the first value of $y_{199.5}$ for $x_{199.5}=2.173828125$ | | | | |
| | 265.8370992402 by 2^{+1}_{-1} | 267.8947337420 by 5^{+1}_{-4} | | |
| | 265.7764162909 by 6^{+1}_{-5} | 265.7764162093 by 7^{+1}_{-6} | | |
| | 265.7764162053 by 8^{+1}_{-7} | 265.7764162051 by 9^{+1}_{-8} | | |
| 201 | 2.177734375 | 276.688024571403 | 235834 | 2857.623981854150 |
| 202 | 2.1796875 | 282.333660403766 | 240648 | 2923.784501169965 |
| 203 | 2.181640625 | 288.110156197960 | 245570 | 2991.632236388510 |
| 204 | 2.18359375 | 294.020852581822 | 250537 | 3061.213784808622 |
| 205 | 2.185546875 | 300.069182532578 | 255762 | 3132.577121718278 |
| 206 | 2.1875 | 306.258674109554 | 261039 | 3205.771643663851 |
| 207 | 2.189453125 | 312.592953274205 | 266435 | 3280.848213168230 |

Table 5 ($y' = x^3y$, nine nodes, 4 times) 1/7

| S | x | y | E | y' |
|-----|---------|----------------|-----|-----------------|
| 1 | -0.25 | 1.000977039492 | 0 | -0.015640266242 |
| 2 | -0.1875 | 1.000309038221 | 0 | -0.006593833992 |
| 3 | -0.125 | 1.000061037019 | 0 | -0.001953244213 |
| 4 | -0.0625 | 1.000003814705 | 0 | -0.000244141556 |
| 5 | 0 | 1.000000000000 | 0 | 0.000000000000 |
| 6 | 0.0625 | 1.000003814705 | 0 | 0.000244141556 |
| 7 | 0.125 | 1.000061037019 | 0 | 0.001953244213 |
| 8 | 0.1875 | 1.000309038221 | 0 | 0.006593833992 |
| 9 | 0.25 | 1.000977039492 | 0 | 0.015640266242 |
| 10 | 0.3125 | 1.002387030245 | 23 | 0.030590424507 |
| 11 | 0.375 | 1.004956088681 | 46 | 0.052995731239 |
| 12 | 0.4375 | 1.009201161005 | 71 | 0.084510741754 |
| 13 | 0.5 | 1.015747708689 | 102 | 0.126968463586 |

the first approximate value of $y_{9.5}$ for $x_{9.5} = -0.28125$

1.0015652217 by 5_{-1}^{+4} 1.0015654581 by 6_{-2}^{+4}
 1.0015654846 by 7_{-3}^{+4} 1.0015654880 by 8_{-4}^{+4}
 1.0015654884 by 9_{-5}^{+4} 1.0015654884 by 10_{-6}^{+4}
 1.0015654884 by 11_{-7}^{+4} 1.0015654884 by 13_{-9}^{+4}

the first approximate value of $y_{10.5}$ for $x_{10.5} = -0.34375$

1.0034969102 by 5_{-2}^{+3} 1.0034968089 by 6_{-3}^{+3}
 1.0034967900 by 7_{-4}^{+3} 1.0034967865 by 8_{-5}^{+3}
 1.0034967860 by 9_{-6}^{+3} 1.0034967860 by 10_{-7}^{+3}
 1.0034967860 by 11_{-8}^{+3} 1.0034967860 by 13_{-10}^{+3}

the first approximate value of $y_{11.5}$ for $x_{11.5} = -0.40625$

1.0068325751 by 5_{-3}^{+2} 1.0068326764 by 6_{-4}^{+2}
 1.0068327029 by 7_{-5}^{+2} 1.0068327091 by 8_{-6}^{+2}
 1.0068327101 by 9_{-7}^{+2} 1.0068327102 by 10_{-8}^{+2}
 1.0068327102 by 11_{-9}^{+2} 1.0068327103 by 13_{-11}^{+2}

the first approximate value of $y_{12.5}$ for $x_{12.5} = -0.46875$

1.0121434202 by 5_{-4}^{+1} 1.0121431838 by 6_{-5}^{+1}
 1.0121431044 by 7_{-6}^{+1} 1.0121430817 by 8_{-7}^{+1}
 1.0121430771 by 9_{-8}^{+1} 1.0121430767 by 10_{-9}^{+1}
 1.0121430765 by 11_{-10}^{+1} 1.0121430764 by 13_{-12}^{+1}

| | | | | |
|----|---------|----------------|-----|----------------|
| 14 | 0.53125 | 1.020112543721 | 47 | 0.152948392557 |
| 15 | 0.5625 | 1.025344064331 | 24 | 0.182489214574 |
| 16 | 0.59375 | 1.031558688000 | 48 | 0.215925935089 |
| 17 | 0.625 | 1.038883909234 | 47 | 0.253633766903 |
| 18 | 0.65625 | 1.047459683459 | 48 | 0.296036502945 |
| 19 | 0.6875 | 1.057440095165 | 74 | 0.343616398112 |
| 20 | 0.71875 | 1.068995362258 | 50 | 0.396925859759 |
| 21 | 0.75 | 1.082314239135 | 106 | 0.456601319635 |
| 22 | 0.78125 | 1.097606892423 | 53 | 0.523379751407 |
| 23 | 0.8125 | 1.115108342808 | 28 | 0.598118415319 |
| 24 | 0.84375 | 1.135082582440 | 56 | 0.681818556829 |
| 25 | 0.875 | 1.157827506807 | 54 | 0.775653974286 |

the first value of $y_{21.5}$ for $x_{21.5} = -0.765625$

1.0896996413 by 5_{-1}^{+4} 1.0896997762 by 6_{-2}^{+4}
 1.0896997817 by 7_{-3}^{+4} 1.0896997821 by 8_{-4}^{+4}
 1.0896997821 by 9_{-5}^{+4} 1.0896997821 by 10_{-6}^{+4}

Table 5 ($y' = x^3y$, nine nodes, 4 times) 2/7

| S | x | y | E | y' |
|---|---------------------------------|--------------------------------|-----|------|
| 1 | 0.0896997821 by 11_{-7}^{+4} | 1.0896997821 by 13_{-9}^{+4} | | |
| the first value of $y_{22.5}$ for $x_{22.5} = 0.796875$ | | | | |
| 1.1060658131 by 5_{-2}^{+3} | 1.1060657553 by 6_{-3}^{+3} | | | |
| 1.1060657513 by 7_{-3}^{+3} | 1.1060657510 by 8_{-5}^{+3} | | | |
| 1.1060657509 by 9_{-6}^{+3} | 1.1060657591 by 10_{-7}^{+3} | | | |
| 1.1060657509 by 11_{-8}^{+3} | 1.1060657509 by 13_{-10}^{+3} | | | |
| the first value of $y_{23.5}$ for $x_{23.5} = 0.828125$ | | | | |
| 1.1247685401 by 5_{-3}^{+2} | 1.1247685979 by 6_{-4}^{+2} | | | |
| 1.1247686035 by 7_{-5}^{+2} | 1.1247686041 by 8_{-6}^{+2} | | | |
| 1.1247686042 by 9_{-7}^{+2} | 1.1247686042 by 10_{-8}^{+2} | | | |
| 1.1247686042 by 11_{-9}^{+2} | 1.1247686043 by 13_{-11}^{+2} | | | |
| the first value of $y_{24.5}$ for $x_{24.5} = 0.859375$ | | | | |
| 1.1460887662 by 5_{-4}^{+1} | 1.1460886314 by 6_{-5}^{+1} | | | |
| 1.1460886147 by 7_{-6}^{+1} | 1.1460886124 by 8_{-7}^{+1} | | | |
| 1.1460886120 by 9_{-8}^{+1} | 1.1460886119 by 10_{-9}^{+1} | | | |
| 1.1460886117 by 11_{-10}^{+1} | 1.1460886112 by 13_{-12}^{+1} | | | |

| S | x | y | E | y' |
|-----|----------|----------------|-----|----------------|
| 26 | 0.890625 | 1.170342770906 | 31 | 0.826794772234 |
| 27 | 0.90625 | 1.183680830269 | 55 | 0.881005608198 |
| 28 | 0.921875 | 1.197891286240 | 32 | 0.938498361498 |
| 29 | 0.9375 | 1.213027194716 | 30 | 0.999503608927 |
| 30 | 0.953125 | 1.229145372318 | 33 | 1.064272482888 |
| 31 | 0.96875 | 1.246306733512 | 59 | 1.133078732241 |
| 32 | 0.984375 | 1.264576661423 | 34 | 1.206221010051 |
| 33 | 1.0 | 1.284025416745 | 57 | 1.284025416745 |
| 34 | 1.015625 | 1.304728587953 | 36 | 1.366848329416 |
| 35 | 1.03125 | 1.326767588928 | 59 | 1.455079554544 |
| 36 | 1.046875 | 1.350230207905 | 39 | 1.549145843583 |
| 37 | 1.0625 | 1.375211215160 | 34 | 1.649514819356 |
| 38 | 1.078125 | 1.401813035071 | 40 | 1.756699364998 |
| 39 | 1.09375 | 1.430146491036 | 64 | 1.871262536718 |
| 40 | 1.109375 | 1.460331631539 | 42 | 1.993823068907 |
| 41 | 1.125 | 1.492498647667 | 63 | 2.125061551073 |
| 42 | 1.140625 | 1.526788892423 | 45 | 2.265727365737 |
| 43 | 1.15625 | 1.563356015712 | 65 | 2.416646492428 |
| 44 | 1.171875 | 1.602367228148 | 50 | 2.578730294704 |
| 45 | 1.1875 | 1.644004711291 | 41 | 2.752985428404 |
| 46 | 1.203125 | 1.688467192058 | 52 | 2.940525026672 |
| 47 | 1.21875 | 1.735971703186 | 73 | 3.142581343423 |
| 48 | 1.234375 | 1.786755553954 | 54 | 3.360520063652 |
| 49 | 1.25 | 1.841078539268 | 75 | 3.595856522008 |
| 50 | 1.265625 | 1.899225418390 | 59 | 3.850274107263 |
| 51 | 1.28125 | 1.961508700983 | 77 | 4.125645177626 |
| 52 | 1.296875 | 2.028271781393 | 67 | 4.424054859418 |
| 53 | 1.3125 | 2.099892470599 | 54 | 4.747828166558 |
| 54 | 1.328125 | 2.176786980794 | 71 | 5.099560945817 |

the first value of $y_{50.5}$ for $x_{50.5} = 1.2734375$

1.9298291981 by 5_{-1}^{+4} 1.9298293071 by 6_{-2}^{+4}
 1.9298293101 by 7_{-3}^{+4} 1.9298293103 by 8_{-4}^{+4}
 1.9298293103 by 9_{-5}^{+4} 1.9298293103 by 10_{-6}^{+4}
 1.9298293103 by 11_{-7}^{+4} 1.9298293103 by 13_{-9}^{+4}

Table 5 ($y' = x^3y$, nine nodes, 4 times) 3/7

| S =step | $E=(\text{error of } y) \times 10^{12}$ | | | |
|--|---|----------------|-----|-----------------|
| | | | | |
| the first value of $y_{51.5}$ for $x_{51.5}=1.2890625$ | | | | |
| 1.9943075533 by 5_{-2}^{+3} | 1.9943075066 by 6_{-3}^{+3} | | | |
| 1.9943075044 by 7_{-4}^{+3} | 1.9943075043 by 8_{-5}^{+3} | | | |
| 1.9943075043 by 9_{-6}^{+3} | 1.9943075043 by 10_{-7}^{+3} | | | |
| 1.9943075043 by 11_{-8}^{+3} | 1.9943075043 by 13_{-10}^{+3} | | | |
| the first value of $y_{52.5}$ for $x_{52.5}=1.3046875$ | | | | |
| 2.0634498117 by 5_{-3}^{+2} | 2.0634498584 by 6_{-4}^{+2} | | | |
| 2.0634498615 by 7_{-5}^{+2} | 2.0634498617 by 8_{-6}^{+2} | | | |
| 2.0634498617 by 9_{-7}^{+2} | 2.0634498617 by 10_{-8}^{+2} | | | |
| 2.0634498617 by 11_{-9}^{+2} | 2.0634498616 by 13_{-11}^{+2} | | | |
| the first value of $y_{53.5}$ for $x_{53.5}=1.3203125$ | | | | |
| 2.1376529848 by 5_{-4}^{+1} | 2.1376528758 by 6_{-5}^{+1} | | | |
| 2.1376528665 by 7_{-6}^{+1} | 2.1376528656 by 8_{-7}^{+1} | | | |
| 2.1376528656 by 9_{-8}^{+1} | 2.1376528656 by 10_{-9}^{+1} | | | |
| 2.1376528657 by 11_{-10}^{+1} | 2.1376528660 by 13_{-12}^{+1} | | | |
| S | x | y | E | y' |
| 55 | 1.3359375 | 2.217353581484 | 72 | 5.286805996431 |
| 56 | 1.34375 | 2.259414427171 | 87 | 5.482155238681 |
| 57 | 1.3515625 | 2.303034446177 | 75 | 5.686025907304 |
| 58 | 1.359375 | 2.348281921291 | 77 | 5.898859748900 |
| 59 | 1.3671875 | 2.395228687321 | 79 | 6.121124623351 |
| 60 | 1.375 | 2.443950341782 | 65 | 6.353316220530 |
| 61 | 1.3828125 | 2.494526469487 | 82 | 0.595959900844 |
| 62 | 1.390625 | 2.547040881816 | 83 | 0.849612668659 |
| 63 | 1.3984375 | 2.601581872261 | 84 | 7.114865290485 |
| 64 | 1.40625 | 2.658242488905 | 101 | 7.392344567916 |
| 65 | 1.4140625 | 2.717120825311 | 89 | 7.682715778256 |
| 66 | 1.421875 | 2.778320331378 | 91 | 7.986685296771 |
| 67 | 1.4296875 | 2.841950145144 | 94 | 8.305003413755 |
| 68 | 1.4375 | 2.908125447620 | 80 | 8.638467363572 |
| 69 | 1.4453125 | 2.976967842178 | 98 | 8.987924582354 |
| 70 | 1.453125 | 3.048605760110 | 99 | 9.354276212253 |
| 71 | 1.4609375 | 3.123174894925 | 101 | 9.738480874262 |
| 72 | 1.46875 | 3.200818667009 | 119 | 10.141558730007 |
| 73 | 1.4765625 | 3.281688721224 | 107 | 10.564595857261 |
| 74 | 1.484375 | 3.365945460231 | 111 | 11.008748966086 |
| 75 | 1.4921875 | 3.453758615755 | 115 | 11.475250482426 |
| 76 | 1.5 | 3.545307861324 | 100 | 11.965414031967 |
| 77 | 1.5078125 | 3.640783469539 | 120 | 12.480640357577 |
| 78 | 1.515625 | 3.740387017131 | 122 | 13.022423706383 |
| 79 | 1.5234375 | 3.844332142242 | 124 | 13.592358728985 |
| 80 | 1.53125 | 3.952845357496 | 145 | 14.192147932862 |
| 81 | 1.5390625 | 4.066166923617 | 132 | 14.823609738978 |
| 82 | 1.546875 | 4.184551788779 | 137 | 15.488687195209 |
| 83 | 1.5546875 | 4.308270598404 | 143 | 16.189457401997 |
| 84 | 1.5625 | 4.437610781849 | 128 | 16.928141715428 |
| 85 | 1.5703125 | 4.572877722070 | 151 | 17.707116795884 |
| 86 | 1.578125 | 4.714396014865 | 154 | 18.528926576659 |
| 87 | 1.5859375 | 4.862510825888 | 156 | 19.396295237865 |
| 88 | 1.59375 | 5.017589352958 | 181 | 20.312141273780 |
| 89 | 1.6015625 | 5.180022402854 | 167 | 21.279592754072 |
| 90 | 1.609375 | 5.350226092650 | 175 | 22.302003889248 |

Table 5 ($y' = x^3y$, nine nodes, 4 times) 4/7

| S =step | $E=(\text{error of } y) \times 10^{12}$ | | | |
|---|---|-----------------|-----|-----------------|
| S | x | y | E | y' |
| 91 | 1.6171875 | 5.528643685532 | 184 | 23.382973017329 |
| 92 | 1.625 | 5.715747573399 | 169 | 24.526362146013 |
| 93 | 1.6328125 | 5.912041418699 | 196 | 25.736318193879 |
| 94 | 1.640625 | 6.118062469026 | 200 | 27.017296088052 |
| 95 | 1.6484375 | 6.334384060452 | 203 | 28.374083896343 |
| 96 | 1.65625 | 6.561618325443 | 232 | 29.811830183015 |
| 97 | 1.6640625 | 6.800419123862 | 220 | 31.336073801085 |
| 98 | 1.671875 | 7.051485217313 | 231 | 32.952776356022 |
| 99 | 1.6796875 | 7.315563707920 | 244 | 34.668357594346 |
| 100 | 1.6875 | 7.593453766227 | 229 | 36.489734004066 |
| 101 | 1.6953125 | 7.886010674249 | 261 | 38.424360938464 |
| 102 | 1.703125 | 8.194150212114 | 268 | 40.480278606584 |
| 103 | 1.7109375 | 8.518853420857 | 273 | 42.666162315837 |
| 104 | 1.71875 | 8.861171775349 | 309 | 44.991377384144 |
| 105 | 1.7265625 | 9.222232805946 | 298 | 47.466039188871 |
| 106 | 1.734375 | 9.603246211390 | 314 | 50.101078870122 |
| 107 | 1.7421875 | 10.005510508398 | 333 | 52.908315254253 |
| 108 | 1.75 | 10.430420269860 | 320 | 55.900533633778 |
| 109 | 1.7578125 | 10.879474007562 | 360 | 59.091572102256 |
| 110 | 1.765625 | 11.354282761001 | 372 | 62.496416217806 |
| 111 | 1.7734375 | 11.856579461636 | 381 | 66.131302861619 |
| 112 | 1.78125 | 12.388229147145 | 427 | 70.013834242161 |
| the first value of $y_{108.5}$ for $x_{108.5}=1.75390625$ | | | | |
| 10.6518309096 by 5_{-1}^{+4} | 10.6518311889 by 6_{-2}^{+4} | | | |
| 10.6518311947 by 7_{-3}^{+4} | 10.6518311948 by 8_{-4}^{+4} | | | |
| 10.6518311949 by 9_{-5}^{+4} | 10.6518311949 by 10_{-6}^{+4} | | | |
| 10.6518311949 by 11_{-7}^{+4} | 10.6518311949 by 13_{-9}^{+4} | | | |
| the first value of $y_{109.5}$ for $x_{109.5}=1.76171875$ | | | | |
| 11.1135537953 by 5_{-2}^{+3} | 11.1135536756 by 6_{-3}^{+3} | | | |
| 11.1135536715 by 7_{-4}^{+3} | 11.1135536713 by 8_{-5}^{+3} | | | |
| 11.1135536713 by 9_{-6}^{+3} | 11.1135536713 by 10_{-7}^{+3} | | | |
| 11.1135536713 by 11_{-8}^{+3} | 11.1135536713 by 13_{-10}^{+3} | | | |
| the first value of $y_{110.5}$ for $x_{110.5}=1.76953125$ | | | | |
| 11.6018816493 by 5_{-3}^{+2} | 11.6018817690 by 6_{-4}^{+2} | | | |
| 11.6018817748 by 7_{-5}^{+2} | 11.6018817751 by 8_{-6}^{+2} | | | |
| 11.6018817752 by 9_{-7}^{+2} | 11.6018817751 by 10_{-8}^{+2} | | | |
| 11.6018817751 by 11_{-9}^{+2} | 11.6018817751 by 13_{-11}^{+2} | | | |
| the first value of $y_{111.5}$ for $x_{111.5}=1.77734375$ | | | | |
| 12.1186134560 by 5_{-4}^{+1} | 12.1186131767 by 6_{-5}^{+1} | | | |
| 12.1186131593 by 7_{-6}^{+1} | 12.1186131581 by 8_{-7}^{+1} | | | |
| 12.1186131580 by 9_{-8}^{+1} | 12.1186131581 by 10_{-9}^{+1} | | | |
| 12.1186131581 by 11_{-10}^{+1} | 12.1186131582 by 13_{-12}^{+1} | | | |
| 113 | 1.78515625 | 12.665683029097 | 415 | 72.053871296007 |
| 114 | 1.7890625 | 12.951240109358 | 428 | 74.163103108235 |
| 115 | 1.79296875 | 13.245175804836 | 435 | 76.344171740772 |
| 116 | 1.796875 | 13.547776068407 | 443 | 78.599830352928 |
| 117 | 1.80078125 | 13.859337832670 | 455 | 80.932948272376 |
| 118 | 1.8046875 | 14.180169474028 | 457 | 83.346516315049 |
| 119 | 1.80859375 | 14.510591298125 | 476 | 85.843652367134 |
| 120 | 1.8125 | 14.850936047483 | 508 | 88.427607241715 |

Table 5 ($y' = x^3y$, nine nodes, 4 times) 5/7

| S=step | | E=(error of y) $\times 10^{12}$ | | |
|--------|------------|-------------------------------------|------|------------------|
| S | x | y | E | y' |
| 121 | 1.81640625 | 15.201549432643 | 498 | 91.101770825626 |
| 122 | 1.8203125 | 15.562790688093 | 514 | 93.869678532670 |
| 123 | 1.82421875 | 15.935033153633 | 524 | 96.735018075810 |
| 124 | 1.828125 | 16.318664883210 | 533 | 99.701636579868 |
| 125 | 1.83203125 | 16.714089282049 | 550 | 102.773548049611 |
| 126 | 1.8359375 | 17.121725773691 | 553 | 105.954941213246 |
| 127 | 1.83984375 | 17.542010498556 | 576 | 109.250187762131 |
| 128 | 1.84375 | 17.975397045309 | 611 | 112.663851006117 |
| 129 | 1.84765625 | 18.442357216978 | 604 | 116.200694968585 |
| 130 | 1.8515625 | 18.883381833682 | 623 | 119.865693945507 |
| 131 | 1.85546875 | 19.358981573239 | 637 | 123.664042549993 |
| 132 | 1.859375 | 19.849687852367 | 649 | 127.601166273525 |
| 133 | 1.86328125 | 20.356053750036 | 671 | 131.682732588715 |
| 134 | 1.8671875 | 20.878654975265 | 676 | 135.914662624007 |
| 135 | 1.87109375 | 21.418090881851 | 703 | 140.303143442923 |
| 136 | 1.875 | 21.974985532078 | 743 | 144.854640958525 |
| 137 | 1.87890625 | 22.549988812287 | 740 | 149.575913520419 |
| 138 | 1.8828125 | 23.14377603062 | 764 | 154.474026211830 |
| 139 | 1.88671875 | 23.757057006313 | 783 | 159.556365892280 |
| 140 | 1.890625 | 24.390561633013 | 797 | 164.830657032552 |
| 141 | 1.89453125 | 25.045056954250 | 826 | 170.304978382635 |
| 142 | 1.8984375 | 25.721340719044 | 835 | 175.987780519903 |
| 143 | 1.90234375 | 26.420244442705 | 868 | 181.887904328763 |
| 144 | 1.90625 | 27.142634969021 | 914 | 188.014600460920 |
| 145 | 1.91015625 | 27.889416110607 | 915 | 194.377549834523 |
| 146 | 1.9140625 | 28.661530371626 | 946 | 200.986885231223 |
| 147 | 1.91796875 | 29.459960756697 | 971 | 207.853214049336 |
| 148 | 1.921875 | 30.285732671430 | 990 | 214.987642284722 |
| 149 | 1.92578125 | 31.139915919002 | 1028 | 222.401799805725 |
| 150 | 1.9296875 | 32.023626798018 | 1042 | 230.107866996817 |
| 151 | 1.93359375 | 32.938030307510 | 1082 | 238.118602852123 |
| 152 | 1.9375 | 33.884342464320 | 1136 | 246.447374598280 |
| 153 | 1.94140625 | 34.863832739508 | 1144 | 255.108188938684 |
| 154 | 1.9453125 | 35.877826620291 | 1184 | 264.115725013200 |
| 155 | 1.94921875 | 36.927708303799 | 1218 | 273.485369168628 |
| 156 | 1.953125 | 38.014923530777 | 1244 | 283.233251651948 |
| 157 | 1.95703125 | 39.140982566393 | 1293 | 293.376285334753 |
| 158 | 1.9609375 | 40.307463336371 | 1314 | 303.932206588430 |
| 159 | 1.96484375 | 41.516014727496 | 1365 | 314.919618439974 |
| 160 | 1.96875 | 42.768360060997 | 1430 | 326.358036138062 |
| 161 | 1.97265625 | 44.066300749074 | 1446 | 338.267935276563 |
| 162 | 1.9765625 | 45.411720144816 | 1498 | 350.670802627386 |
| 163 | 1.98046875 | 46.806587595761 | 1544 | 363.589189839109 |
| 164 | 1.984375 | 48.252962713543 | 1580 | 377.046770179956 |
| 165 | 1.98828125 | 49.752999871256 | 1644 | 391.068398503019 |
| 166 | 1.9921875 | 51.308952941537 | 1676 | 405.680174627768 |
| 167 | 1.99609375 | 52.923180289587 | 1740 | 420.909510348022 |
| 168 | 2.0 | 54.598150034965 | 1821 | 436.785200279719 |
| 169 | 2.00390625 | 56.336445598227 | 1849 | 453.337496786789 |
| 170 | 2.0078125 | 58.140771548814 | 1917 | 470.598189233352 |
| 171 | 2.01171875 | 60.013959770862 | 1979 | 488.600687820720 |
| 172 | 2.015625 | 61.958975966408 | 2030 | 507.380112298399 |
| 173 | 2.01953125 | 63.978926514947 | 2113 | 526.973385843283 |
| 174 | 2.0234375 | 66.077065710140 | 2162 | 547.419334425730 |

Table 5 ($y' = x^3y$, nine nodes, 4 times) 6/7

| S=step | | E=(error of y) $\times 10^{12}$ | | |
|--------|------------|-------------------------------------|------|-------------------|
| S | x | y | E | y' |
| 175 | 2.02734375 | 68.256803396361 | 2244 | 568.758792006782 |
| 176 | 2.03125 | 70.521713027656 | 2346 | 591.034711920781 |
| 177 | 2.03515625 | 72.875540174756 | 2393 | 614.292284834390 |
| 178 | 2.0390625 | 75.322211506578 | 2484 | 638.579063692249 |
| 179 | 2.04296875 | 77.865844273578 | 2568 | 663.945096079993 |
| 180 | 2.046875 | 80.510756323935 | 2639 | 690.443064479951 |
| 181 | 2.05078125 | 83.261476683624 | 2750 | 718.128434910476 |
| 182 | 2.0546875 | 86.122756734137 | 2822 | 747.059614478559 |
| 183 | 2.05859375 | 89.099582024466 | 2930 | 777.298118416693 |
| 184 | 2.0625 | 92.197184754514 | 3062 | 808.908747197992 |
| 185 | 2.06640625 | 95.421056971368 | 3134 | 841.959774379499 |
| 186 | 2.0703125 | 98.776964521640 | 3257 | 876.523145859729 |
| 187 | 2.07421875 | 102.270961804991 | 3373 | 912.674691275060 |
| 188 | 2.078125 | 105.909407379009 | 3473 | 950.494348327369 |
| 189 | 2.08203125 | 109.698980466743 | 3623 | 990.066400870575 |
| 190 | 2.0859375 | 113.646698422343 | 3727 | 1031.479731646882 |
| 191 | 2.08984375 | 117.759935214672 | 3873 | 1074.828090631866 |
| 192 | 2.09375 | 122.046440990635 | 4047 | 1120.210379994700 |
| 193 | 2.09765625 | 126.514362786165 | 4156 | 1167.730956768171 |
| 194 | 2.1015625 | 131.172266456246 | 4325 | 1217.499954389514 |
| 195 | 2.10546875 | 136.029159899102 | 4485 | 1269.633624344008 |
| 196 | 2.109375 | 141.094517657020 | 4630 | 1324.254699250755 |
| 197 | 2.11328125 | 146.378306979411 | 4833 | 1381.492778801152 |
| 198 | 2.1171875 | 151.891015440235 | 4986 | 1441.484740066743 |
| 199 | 2.12109375 | 157.643680209085 | 5185 | 1504.375173808385 |
| 200 | 2.125 | 163.647919079462 | 5419 | 1570.316848510543 |
| 201 | 2.12890625 | 169.915963367203 | 5583 | 1639.471204008648 |
| 202 | 2.1328125 | 176.460692798454 | 5817 | 1712.008876698609 |
| 203 | 2.13671875 | 183.295672513563 | 6043 | 1788.110258446908 |
| 204 | 2.140625 | 190.435192324499 | 6252 | 1867.966091496187 |
| 205 | 2.14453125 | 197.894308370084 | 6532 | 1951.778101798376 |
| 206 | 2.1484375 | 205.688887324138 | 6757 | 2039.759673390002 |
| 207 | 2.15234375 | 213.835653323361 | 7035 | 2132.136566622779 |
| 208 | 2.15625 | 222.352237790481 | 7355 | 2229.147683237091 |

the first value of $y_{204.5}$ for $x_{204.5}=2.142578125$

194.1238273014 by 5_{-1}^{+4} 194.1238286638 by 6_{-2}^{+4}
 194.1238286836 by 7_{-3}^{+4} 194.1238286840 by 8_{-4}^{+4}
 194.1238286840 by 9_{-5}^{+4} 194.1238286840 by 10_{-6}^{+4}
 194.1238286840 by 11_{-7}^{+4} 194.1238286840 by 13_{-9}^{+4}

the first value of $y_{205.5}$ for $x_{205.5}=2.146484375$

201.7486409895 by 5_{-2}^{+3} 201.7486404057 by 6_{-3}^{+3}
 201.7486403915 by 7_{-4}^{+3} 201.7486403911 by 8_{-5}^{+3}
 201.7486403911 by 9_{-6}^{+3} 201.7486403911 by 10_{-7}^{+3}
 201.7486403911 by 11_{-8}^{+3} 201.7486403911 by 13_{-10}^{+3}

the first value of $y_{206.5}$ for $x_{206.5}=2.150390625$

209.7171662437 by 5_{-3}^{+2} 209.7171668276 by 6_{-4}^{+2}
 209.7171668474 by 7_{-5}^{+2} 209.7171668481 by 8_{-6}^{+2}
 209.7171668481 by 9_{-7}^{+2} 209.7171668481 by 10_{-8}^{+2}
 209.7171668481 by 11_{-9}^{+2} 209.7171668482 by 13_{-11}^{+2}

the first value of $y_{207.5}$ for $x_{207.5}=2.154296875$

218.0465808437 by 5_{-4}^{+1} 218.0465794813 by 6_{-5}^{+1}

Table 5 ($y' = x^3y$, nine nodes, 4 times) 7/7

$S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

.....
 218.0465794219 by 7_{-6}^{+1} 218.0465794191 by 8_{-7}^{+1}
 218.0465794191 by 9_{-8}^{+1} 218.0465794190 by 10_{-9}^{+1}
 218.0465794191 by 11_{-10}^{+1} 218.0465794190 by 13_{-12}^{+1}

| S | x | y | E | y' |
|-----|-------------|------------------|-------|-------------------|
| 209 | 2.158203125 | 226.754982809116 | 7451 | 2279.469524974923 |
| 210 | 2.16015625 | 231.257232343083 | 7628 | 2331.045881508595 |
| 211 | 2.162109375 | 235.861469593980 | 7753 | 2383.910683334526 |
| 212 | 2.1640625 | 240.570244990158 | 7902 | 2438.098844916540 |
| 213 | 2.166015625 | 245.386178137710 | 8066 | 2493.646295018427 |
| 214 | 2.16796875 | 250.311959832025 | 8233 | 2550.590008028089 |
| 215 | 2.169921875 | 255.350354131453 | 8392 | 2608.968036302621 |
| 216 | 2.171875 | 260.504200495230 | 8610 | 2668.819543570702 |
| 217 | 2.173828125 | 265.776415987333 | 8736 | 2730.184839424230 |
| 218 | 2.17578125 | 271.169997549643 | 8940 | 2793.105414948882 |
| 219 | 2.177734375 | 276.688024344667 | 9098 | 2857.623979512433 |
| 220 | 2.1796875 | 282.333660172391 | 9273 | 2923.784498773892 |
| 221 | 2.181640625 | 288.110155961865 | 9475 | 2991.632233936977 |
| 222 | 2.18359375 | 294.020852340954 | 9669 | 3061.213782300813 |
| 223 | 2.185546875 | 300.069182286680 | 9865 | 3132.577119151230 |
| 224 | 2.1875 | 306.258673858630 | 10115 | 3205.771641037294 |
| 225 | 2.189453125 | 312.592953018048 | 10277 | 3280.848210479708 |
| 226 | 2.19140625 | 319.075746536431 | 10515 | 3357.859202171168 |
| 227 | 2.193359375 | 325.710884994765 | 10714 | 3436.858550701428 |
| 228 | 2.1953125 | 332.502305878536 | 10921 | 3517.901799882648 |
| 229 | 2.197265625 | 339.454056770064 | 11167 | 3601.046153713495 |
| 230 | 2.19921875 | 346.570298642143 | 11396 | 3686.350529047233 |
| 231 | 2.201171875 | 353.855309256485 | 11635 | 3773.875610024782 |
| 232 | 2.203125 | 361.313486670027 | 11926 | 3863.683904329837 |
| 233 | 2.205078125 | 368.949352852810 | 12132 | 3955.839801331201 |
| 234 | 2.20703125 | 376.767557421993 | 12412 | 4050.409632187493 |
| 235 | 2.208984375 | 384.772881494169 | 12659 | 4147.461731964888 |
| 236 | 2.2109375 | 392.970241662038 | 12908 | 4247.066503861083 |
| 237 | 2.212890625 | 401.364694098059 | 13206 | 4349.296485593286 |
| 238 | 2.21484375 | 409.961438789963 | 13481 | 4454.226418033127 |
| 239 | 2.216796875 | 418.765823912934 | 13772 | 4561.933316172079 |
| 240 | 2.21875 | 427.783350342384 | 14114 | 4672.496542492458 |
| 241 | 2.220703125 | 437.019676312407 | 14372 | 4785.997882833132 |
| 242 | 2.22265625 | 446.480622225553 | 14706 | 4902.521624846278 |
| 243 | 2.224609375 | 456.172175617398 | 15010 | 5022.154639119456 |
| 244 | 2.2265625 | 466.100496283215 | 15312 | 5144.986463080497 |
| 245 | 2.228515625 | 476.271921570814 | 15673 | 5271.109387768614 |
| 246 | 2.23046875 | 486.692971845648 | 16004 | 5400.618547579279 |
| 247 | 2.232421875 | 497.370356134686 | 16359 | 5533.612013096088 |
| 248 | 2.234375 | 508.310977954227 | 16764 | 5670.190887110126 |
| 249 | 2.236328125 | 519.521941328492 | 17086 | 5810.459403947462 |
| 250 | 2.23828125 | 531.010557006151 | 17487 | 5954.525032230561 |
| 251 | 2.240234375 | 542.784348849930 | 17862 | 6102.498581179419 |
| 252 | 2.2421875 | 554.851060618580 | 18230 | 6254.494310603306 |
| 253 | 2.244140625 | 567.218662516406 | 18668 | 6410.630044700744 |
| 254 | 2.24609375 | 579.895358569398 | 19070 | 6571.027289809532 |
| 255 | 2.248046875 | 592.889593785414 | 19502 | 6735.811356259309 |

Table 6 ($y' = x^3y$, nine nodes, 5 times) 1/7

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{12}$ | | |
|--|---------|---|-----|-----------------|
| S | x | y | E | y' |
| 1 | -0.25 | 1.000977039492 | 0 | -0.015640266242 |
| 2 | -0.1875 | 1.000309038221 | 0 | -0.006593833992 |
| 3 | -0.125 | 1.000061037019 | 0 | -0.001953244213 |
| 4 | -0.0625 | 1.000003814705 | 0 | -0.000244141556 |
| 5 | 0 | 1.000000000000 | 0 | 0.000000000000 |
| 6 | 0.0625 | 1.000003814705 | 0 | 0.000244141556 |
| 7 | 0.125 | 1.000061037019 | 0 | 0.001953244213 |
| 8 | 0.1875 | 1.000309038221 | 0 | 0.006593833992 |
| 9 | 0.25 | 1.000977039492 | 0 | 0.015640266242 |
| 10 | 0.3125 | 1.002387030245 | 23 | 0.030590424507 |
| 11 | 0.375 | 1.004956088681 | 46 | 0.052995731239 |
| 12 | 0.4375 | 1.009201161005 | 71 | 0.084510741754 |
| 13 | 0.5 | 1.015747708689 | 102 | 0.126968463586 |
| 14 | 0.5625 | 1.025344064453 | 146 | 0.182489214596 |
| 15 | 0.625 | 1.038883909402 | 215 | 0.253633766944 |
| 16 | 0.6875 | 1.057440095421 | 330 | 0.343616398195 |
| the first approximate value of $y_{12.5}$ for $x_{12.5}=0.46875$ | | | | |
| 1.0121420650 by 5_{-1}^{+4} | | 1.0121429899 by 6_{-2}^{+4} | | |
| 1.0121430674 by 7_{-3}^{+4} | | 1.0121430753 by 8_{-4}^{+4} | | |
| 1.0121430762 by 9_{-5}^{+4} | | 1.0121430763 by 10_{-6}^{+4} | | |
| 1.0121430763 by 11_{-7}^{+4} | | 1.0121430763 by 13_{-9}^{+4} | | |
| the first approximate value of $y_{13.5}$ for $x_{13.5}=0.53125$ | | | | |
| 1.0201130048 by 5_{-2}^{+3} | | 1.0201126085 by 6_{-3}^{+3} | | |
| 1.0201125531 by 7_{-4}^{+3} | | 1.0201125452 by 8_{-5}^{+3} | | |
| 1.0201125441 by 9_{-6}^{+3} | | 1.0201125439 by 10_{-7}^{+3} | | |
| 1.0201125438 by 11_{-8}^{+3} | | 1.0201125438 by 13_{-10}^{+3} | | |
| the first approximate value of $y_{14.5}$ for $x_{14.5}=0.59375$ | | | | |
| 1.0315581969 by 5_{-3}^{+2} | | 1.0315585932 by 6_{-4}^{+2} | | |
| 1.0315586708 by 7_{-5}^{+2} | | 1.0315586849 by 8_{-6}^{+2} | | |
| 1.0315586874 by 9_{-7}^{+2} | | 1.0315586879 by 10_{-8}^{+2} | | |
| 1.0315586881 by 11_{-9}^{+2} | | 1.0315586881 by 13_{-11}^{+2} | | |
| the first approximate value of $y_{15.5}$ for $x_{15.5}=0.65625$ | | | | |
| 1.0474609077 by 5_{-4}^{+1} | | 1.0474599828 by 6_{-5}^{+1} | | |
| 1.0474597502 by 7_{-6}^{+1} | | 1.0474596982 by 8_{-7}^{+1} | | |
| 1.0474596876 by 9_{-8}^{+1} | | 1.0474596850 by 10_{-9}^{+1} | | |
| 1.0474596841 by 11_{-10}^{+1} | | 1.0474596837 by 13_{-12}^{+1} | | |
| 17 | 0.71875 | 1.068995362366 | 157 | 0.396925859799 |
| 18 | 0.75 | 1.082314239140 | 111 | 0.456601319637 |
| 19 | 0.78125 | 1.097606892533 | 163 | 0.523379751460 |
| 20 | 0.8125 | 1.115108342938 | 158 | 0.598118415389 |
| 21 | 0.84375 | 1.135082582552 | 168 | 0.681818556896 |
| 22 | 0.875 | 1.157827506988 | 235 | 0.775653974408 |
| 23 | 0.90625 | 1.183680830389 | 175 | 0.881005608287 |
| 24 | 0.9375 | 1.213027195044 | 358 | 0.999503609197 |
| 25 | 0.96875 | 1.246306733651 | 198 | 1.133078732367 |
| 26 | 1.0 | 1.284025416832 | 144 | 1.284025416832 |
| 27 | 1.03125 | 1.326767589089 | 220 | 1.455079554721 |
| 28 | 1.0625 | 1.375211215329 | 203 | 1.649514819559 |
| 29 | 1.09375 | 1.430146491208 | 236 | 1.871262536943 |
| 30 | 1.125 | 1.492498647915 | 311 | 2.125061551426 |

Table 6 ($y' = x^3y$, nine nodes, 5 times) 2/7

| $S=\text{step}$ | $E=(\text{error of } y) \times 10^{12}$ |
|---|---|
| the first value of $y_{25.5}$ for $x_{25.5}=1.015625$ | |
| 1.3047279336 by 5_{-1}^{+4} | 1.3047285572 by 6_{-2}^{+4} |
| 1.3047285859 by 7_{-3}^{+4} | 1.3047285879 by 8_{-4}^{+4} |
| 1.3047285881 by 9_{-5}^{+4} | 1.3047285881 by 10_{-6}^{+4} |
| 1.3047285881 by 11_{-7}^{+4} | 1.3047285881 by 13_{-9}^{+4} |
| the first value of $y_{26.5}$ for $x_{26.5}=1.046875$ | |
| 1.3502304981 by 5_{-2}^{+3} | 1.3502302309 by 6_{-3}^{+3} |
| 1.3502302104 by 7_{-4}^{+3} | 1.3502302084 by 8_{-5}^{+3} |
| 1.3502302081 by 9_{-6}^{+3} | 1.3502302081 by 10_{-7}^{+3} |
| 1.3502302081 by 11_{-8}^{+3} | 1.3502302080 by 13_{-10}^{+3} |
| the first value of $y_{27.5}$ for $x_{27.5}=1.078125$ | |
| 1.4018127350 by 5_{-3}^{+2} | 1.4018130023 by 6_{-4}^{+2} |
| 1.4018130309 by 7_{-5}^{+2} | 1.4018130346 by 8_{-6}^{+2} |
| 1.4018130351 by 9_{-7}^{+2} | 1.4018130352 by 10_{-8}^{+2} |
| 1.4018130353 by 11_{-9}^{+2} | 1.4018130355 by 13_{-11}^{+2} |
| the first value of $y_{28.5}$ for $x_{28.5}=1.109375$ | |
| 1.4603323577 by 5_{-4}^{+1} | 1.4603317341 by 6_{-5}^{+1} |
| 1.4603316481 by 7_{-6}^{+1} | 1.4603316347 by 8_{-7}^{+1} |
| 1.4603316324 by 9_{-8}^{+1} | 1.4603316319 by 10_{-9}^{+1} |
| 1.4603316316 by 11_{-10}^{+1} | 1.4603316299 by 13_{-12}^{+1} |

| S | x | y | E | y' |
|-----|----------|----------------|-----|-----------------|
| 31 | 1.140625 | 1.526788892609 | 231 | 2.265727366012 |
| 32 | 1.15625 | 1.563356015901 | 254 | 2.416646492720 |
| 33 | 1.171875 | 1.602367228343 | 246 | 2.578730295018 |
| 34 | 1.1875 | 1.644004711491 | 241 | 2.752985428739 |
| 35 | 1.203125 | 1.688467192264 | 258 | 2.940525027031 |
| 36 | 1.21875 | 1.735971703395 | 282 | 3.142581343802 |
| 37 | 1.234375 | 1.786755554171 | 271 | 3.306520064060 |
| 38 | 1.25 | 1.841078539560 | 366 | 3.595856522577 |
| 39 | 1.265625 | 1.899225418624 | 293 | 3.850274107736 |
| 40 | 1.28125 | 1.961508701220 | 314 | 4.125645178125 |
| 41 | 1.296875 | 2.028271781646 | 320 | 4.424054859971 |
| 42 | 1.3125 | 2.099892470853 | 308 | 4.747828167131 |
| 43 | 1.328125 | 2.176786981066 | 342 | 5.099560946453 |
| 44 | 1.34375 | 2.259414427447 | 363 | 5.482155239351 |
| 45 | 1.359375 | 2.348281921575 | 362 | 5.898859749614 |
| 46 | 1.375 | 2.443950342179 | 463 | 6.353316221563 |
| 47 | 1.390625 | 2.547040882133 | 400 | 6.849612669512 |
| 48 | 1.40625 | 2.658242489227 | 422 | 7.392344568810 |
| 49 | 1.421875 | 2.778320331735 | 449 | 7.986685297798 |
| 50 | 1.4375 | 2.908125447971 | 431 | 8.638467364614 |
| 51 | 1.453125 | 3.048605760499 | 488 | 9.354276213446 |
| 52 | 1.46875 | 3.200818667403 | 513 | 10.141558731255 |
| 53 | 1.484375 | 3.365945460646 | 525 | 11.008748967441 |
| 54 | 1.5 | 3.545307861867 | 643 | 11.965414033780 |
| 55 | 1.515625 | 3.740387017603 | 594 | 13.022423708027 |
| 56 | 1.53125 | 3.952845357979 | 628 | 14.192147934599 |
| 57 | 1.546875 | 4.184551789328 | 686 | 15.488687197239 |
| 58 | 1.5625 | 4.437610782392 | 671 | 16.928141717498 |
| 59 | 1.578125 | 4.714396015477 | 766 | 18.528926579064 |

Table 6 ($y' = x^3y$, nine nodes, 5 times) 3/7

| $S=\text{step}$ | $E=(\text{error of } y) \times 10^{12}$ | |
|--|---|--|
| the first value of $y_{55.5}$ for $x_{55.5}=1.5234375$ | | |
| 3.8443311899 by 5_{-1}^{+4} | 3.8443321099 by 6_{-2}^{+4} | |
| 3.8443321410 by 7_{-3}^{+4} | 3.8443321426 by 8_{-4}^{+4} | |
| 3.8443321427 by 9_{-5}^{+4} | 3.8443321427 by 10_{-6}^{+4} | |
| 3.8443321427 by 11_{-7}^{+4} | 3.8443321427 by 13_{-9}^{+4} | |
| the first value of $y_{56.5}$ for $x_{56.5}=1.5390625$ | | |
| 4.0661673424 by 5_{-2}^{+3} | 4.0661669482 by 6_{-3}^{+3} | |
| 4.0661669259 by 7_{-4}^{+3} | 4.0661669243 by 8_{-5}^{+3} | |
| 4.0661669242 by 9_{-6}^{+3} | 4.0661669242 by 10_{-7}^{+3} | |
| 4.0661669242 by 11_{-8}^{+3} | 4.0661669242 by 13_{-10}^{+3} | |
| the first value of $y_{57.5}$ for $x_{57.5}=1.5546875$ | | |
| 4.3082701703 by 5_{-3}^{+2} | 4.3082705646 by 6_{-4}^{+2} | |
| 4.3082705957 by 7_{-5}^{+2} | 4.3082705986 by 8_{-6}^{+2} | |
| 4.3082705989 by 9_{-7}^{+2} | 4.3082705989 by 10_{-8}^{+2} | |
| 4.3082705989 by 11_{-9}^{+2} | 4.3082705988 by 13_{-11}^{+2} | |
| the first value of $y_{58.5}$ for $x_{58.5}=1.5703125$ | | |
| 4.5728787481 by 5_{-4}^{+1} | 4.5728778282 by 6_{-5}^{+1} | |
| 4.5728777348 by 7_{-6}^{+1} | 4.5728777242 by 8_{-7}^{+1} | |
| 4.5728777229 by 9_{-8}^{+1} | 4.5728777228 by 10_{-9}^{+1} | |
| 4.5728777230 by 11_{-10}^{+1} | 4.5728777237 by 13_{-12}^{+1} | |

| S | x | y | E | y' |
|-----|-----------|-----------------|------|-----------------|
| 60 | 1.5859375 | 4.862510826524 | 792 | 19.396295240400 |
| 61 | 1.59375 | 5.017589353582 | 804 | 20.312141276305 |
| 62 | 1.6015625 | 5.180022403529 | 843 | 21.279592756846 |
| 63 | 1.609375 | 5.350226093351 | 877 | 22.302003892172 |
| 64 | 1.6171875 | 5.528643686249 | 902 | 23.382973020363 |
| 65 | 1.625 | 5.715747574109 | 879 | 24.526362149058 |
| 66 | 1.6328125 | 5.912041419465 | 962 | 25.736318197214 |
| 67 | 1.640625 | 6.118062469821 | 995 | 27.017296091562 |
| 68 | 1.6484375 | 6.334384061274 | 1026 | 28.374083900028 |
| 69 | 1.65625 | 6.561618326269 | 1058 | 29.811830186764 |
| 70 | 1.6640625 | 6.800419124744 | 1101 | 31.336073805147 |
| 71 | 1.671875 | 7.051485218236 | 1154 | 32.592776360334 |
| 72 | 1.6796875 | 7.315563708865 | 1189 | 34.668357598825 |
| 73 | 1.6875 | 7.593453767181 | 1182 | 36.489734008647 |
| 74 | 1.6953125 | 7.886010675268 | 1280 | 38.424360943427 |
| 75 | 1.703125 | 8.194150213178 | 1332 | 40.480278611838 |
| 76 | 1.7109375 | 8.518853421959 | 1375 | 42.666162321357 |
| 77 | 1.71875 | 8.861171776472 | 1432 | 44.991377389846 |
| 78 | 1.7265625 | 9.222232807137 | 1489 | 47.466039195001 |
| 79 | 1.734375 | 9.603246212643 | 1567 | 50.101078876661 |
| 80 | 1.7421875 | 10.005510509688 | 1623 | 52.908315261074 |
| 81 | 1.75 | 10.430420271180 | 1640 | 55.900533640854 |
| 82 | 1.7578125 | 10.879474008965 | 1763 | 59.091572109874 |
| 83 | 1.765625 | 11.354282762473 | 1843 | 62.496416225904 |
| 84 | 1.7734375 | 11.856579463166 | 1912 | 66.131302870156 |
| 85 | 1.78125 | 12.388229148723 | 2006 | 70.013834251084 |
| 86 | 1.7890625 | 12.951240111018 | 2088 | 74.163103117738 |
| 87 | 1.796875 | 13.547776070169 | 2205 | 78.599830363152 |
| 88 | 1.8046875 | 14.180169475869 | 2298 | 83.346516325870 |
| 89 | 1.8125 | 14.850936049327 | 2352 | 88.427607252697 |
| 90 | 1.8203125 | 15.562790690098 | 2520 | 93.86978544767 |

Table 6 ($y' = x^3y$, nine nodes, 5 times) 4/7

| $S=\text{step}$ | | $E=(\text{error of } y) \times 10^{12}$ | | |
|---|------------|---|----------------------------------|------------------|
| S | x | y | E | y' |
| 91 | 1.828125 | 16.318664885322 | 2645 | 99.701636592772 |
| 92 | 1.8359375 | 17.121725775896 | 2758 | 105.954941226893 |
| 93 | 1.84375 | 17.975397047609 | 2912 | 112.663851020537 |
| 94 | 1.8515625 | 18.883381836101 | 3043 | 119.865693960863 |
| 95 | 1.859375 | 19.849687854942 | 3223 | 127.601166290076 |
| 96 | 1.8671875 | 20.878654977970 | 3381 | 135.914662641615 |
| 97 | 1.875 | 21.974985534835 | 3500 | 144.854640976698 |
| 98 | 1.8828125 | 23.143777606044 | 3745 | 154.474026231731 |
| 99 | 1.890625 | 24.390561636165 | 3949 | 164.830657053854 |
| 100 | 1.8984375 | 25.721340722352 | 4143 | 175.987780542538 |
| 101 | 1.90625 | 27.142634972504 | 4397 | 188.014600485046 |
| 102 | 1.9140625 | 28.661530375299 | 4618 | 200.986885256976 |
| 103 | 1.921875 | 30.285732675349 | 4909 | 214.987642312538 |
| 104 | 1.9296875 | 32.023626802160 | 5184 | 230.107867026576 |
| 105 | 1.9375 | 33.884342468603 | 5420 | 246.447374629431 |
| 106 | 1.9453125 | 35.877826624912 | 5805 | 264.115725047218 |
| 107 | 1.953125 | 38.014923535683 | 6150 | 283.233251688506 |
| 108 | 1.9609375 | 40.307463341552 | 6495 | 303.932206627493 |
| 109 | 1.96875 | 42.768360066493 | 6927 | 326.358036180006 |
| 110 | 1.9765625 | 45.411720150638 | 7320 | 350.670802672344 |
| 111 | 1.984375 | 48.252962719774 | 7811 | 377.046770228647 |
| 112 | 1.9921875 | 51.308952948164 | 8304 | 405.680174680168 |
| 113 | 2.0 | 54.598150041905 | 8760 | 436.785200335237 |
| 114 | 2.0078125 | 58.140771556302 | 9405 | 470.598189293959 |
| 115 | 2.015625 | 61.958975974398 | 10020 | 507.380112363834 |
| the first value of $y_{111.5}$ for $x_{111.5}=1.98828125$ | | | | |
| | | 49.752994755517 by 5_{-1}^{+4} | 49.7529997479 by 6_{-2}^{+4} | |
| | | 49.7529998727 by 7_{-3}^{+4} | 49.7529998774 by 8_{-4}^{+4} | |
| | | 49.7529998777 by 9_{-5}^{+4} | 49.7529998777 by 10_{-6}^{+4} | |
| | | 49.7529998777 by 11_{-7}^{+4} | 49.7529998777 by 13_{-9}^{+4} | |
| the first value of $y_{112.5}$ for $x_{112.5}=1.99609375$ | | | | |
| | | 52.9231825302 by 5_{-2}^{+3} | 52.9231803906 by 6_{-3}^{+3} | |
| | | 52.9231803014 by 7_{-4}^{+3} | 52.9231802967 by 8_{-5}^{+3} | |
| | | 52.9231802964 by 9_{-6}^{+3} | 52.9231802964 by 10_{-7}^{+3} | |
| | | 52.9231802964 by 11_{-8}^{+3} | 52.9231802963 by 13_{-10}^{+3} | |
| the first value of $y_{113.5}$ for $x_{113.5}=2.00390625$ | | | | |
| | | 56.3364433318 by 5_{-3}^{+2} | 56.3364454714 by 6_{-4}^{+2} | |
| | | 56.3364455962 by 7_{-5}^{+2} | 56.3364456047 by 8_{-6}^{+2} | |
| | | 56.3364456054 by 9_{-7}^{+2} | 56.3364456054 by 10_{-8}^{+2} | |
| | | 56.3364456054 by 11_{-9}^{+2} | 56.3364456055 by 13_{-11}^{+2} | |
| the first value of $y_{114.5}$ for $x_{114.5}=2.01171875$ | | | | |
| | | 60.0139651797 by 5_{-4}^{+1} | 60.0139601873 by 6_{-5}^{+1} | |
| | | 60.0139598128 by 7_{-6}^{+1} | 60.0139597818 by 8_{-7}^{+1} | |
| | | 60.0139597790 by 9_{-8}^{+1} | 60.0139597787 by 10_{-9}^{+1} | |
| | | 60.0139597786 by 11_{-10}^{+1} | 60.0139597781 by 13_{-12}^{+1} | |
| 116 | 2.01953125 | 63.978926523111 | 10278 | 526.973385910530 |
| 117 | 2.0234375 | 66.077065718649 | 10672 | 547.419334496229 |
| 118 | 2.02734375 | 68.256803340509 | 10970 | 568.758792079494 |
| 119 | 2.03125 | 70.521713036624 | 11313 | 591.034711995933 |
| 120 | 2.03515625 | 72.875540184074 | 11711 | 614.292284912933 |

Table 6 ($y' = x^3y$, nine nodes, 5 times) 5/7

| S=step | | E=(error of y) $\times 10^{12}$ | | |
|--------|------------|-------------------------------------|-------|-------------------|
| S | x | y | E | y' |
| 121 | 2.0390625 | 75.322211516258 | 12164 | 638.579063774317 |
| 122 | 2.04296875 | 77.865844283521 | 12511 | 663.945096164778 |
| 123 | 2.046875 | 80.510756334299 | 13002 | 690.443064568824 |
| 124 | 2.05078125 | 83.261476694261 | 13387 | 718.128435002218 |
| 125 | 2.0546875 | 86.122756745206 | 13891 | 747.059614574581 |
| 126 | 2.05859375 | 89.099582035870 | 14334 | 777.298118516181 |
| 127 | 2.0625 | 92.197184766246 | 14794 | 808.908747300926 |
| 128 | 2.06640625 | 95.421056983581 | 15347 | 841.959774487264 |
| 129 | 2.0703125 | 98.776964534318 | 15935 | 876.523145972228 |
| 130 | 2.07421875 | 102.270961818061 | 16442 | 912.674691391696 |
| 131 | 2.078125 | 105.909407392624 | 17089 | 950.494348449561 |
| 132 | 2.08203125 | 109.698980480766 | 17646 | 990.066400997138 |
| 133 | 2.0859375 | 113.646698436932 | 18316 | 1031.479731779294 |
| 134 | 2.08984375 | 117.759935229754 | 18955 | 1074.828090769523 |
| 135 | 2.09375 | 122.046441006179 | 19591 | 1120.210380137375 |
| 136 | 2.09765625 | 126.514362802367 | 20358 | 1167.730956917717 |
| 137 | 2.1015625 | 131.172266473068 | 21146 | 1217.499954545646 |
| 138 | 2.10546875 | 136.029159916493 | 21877 | 1269.633624506334 |
| 139 | 2.109375 | 141.094517675142 | 22753 | 1324.254699420846 |
| 140 | 2.11328125 | 146.378306998130 | 23552 | 1381.492778977817 |
| 141 | 2.1171875 | 151.891015459717 | 24468 | 1441.484740251630 |
| 142 | 2.12109375 | 157.643680229281 | 25382 | 1504.375174001117 |
| 143 | 2.125 | 163.647919100324 | 26281 | 1570.316848710725 |
| 144 | 2.12890625 | 169.915963388969 | 27349 | 1639.471204218665 |
| 145 | 2.1328125 | 176.460692821071 | 28434 | 1712.008876918041 |
| 146 | 2.13671875 | 183.295672537004 | 29484 | 1788.110258675584 |
| 147 | 2.140625 | 190.435192348943 | 30697 | 1867.966091735961 |
| 148 | 2.14453125 | 197.894308395396 | 31845 | 1951.778102048027 |
| 149 | 2.1484375 | 205.688887350505 | 33124 | 2039.759673651475 |
| 150 | 2.15234375 | 213.835653350760 | 34433 | 2132.136566895971 |
| 151 | 2.15625 | 222.352237818849 | 35724 | 2229.147683521492 |
| 152 | 2.16015625 | 231.257232372663 | 37228 | 2331.045881806957 |
| 153 | 2.1640625 | 240.570245021010 | 38753 | 2438.098845229211 |
| 154 | 2.16796875 | 250.311959864061 | 40269 | 2550.590008354531 |
| 155 | 2.171875 | 260.504200528599 | 41979 | 2668.819543912564 |
| 156 | 2.17578125 | 271.169997584342 | 43639 | 2793.105415306291 |
| 157 | 2.1796875 | 282.333660208576 | 45458 | 2923.784499148616 |
| 158 | 2.18359375 | 294.020852378633 | 47348 | 3061.213782693108 |
| 159 | 2.1875 | 306.258673897738 | 49223 | 3205.771641446660 |
| 160 | 2.19140625 | 319.075746577285 | 51368 | 3357.859202601094 |
| 161 | 2.1953125 | 332.502305921167 | 53552 | 3517.901800333685 |
| 162 | 2.19921875 | 346.570298686507 | 55760 | 3686.350529519113 |
| 163 | 2.203125 | 361.313486716312 | 58211 | 3863.683904824783 |
| 164 | 2.20703125 | 376.767557470215 | 60635 | 4050.409632705907 |
| 165 | 2.2109375 | 392.970241712393 | 63264 | 4247.066504405305 |
| 166 | 2.21484375 | 409.961438842501 | 66018 | 4454.226418603945 |
| 167 | 2.21875 | 427.783350397047 | 68777 | 4672.496543089523 |
| 168 | 2.22265625 | 446.480622282728 | 71881 | 4902.521625474080 |
| 169 | 2.2265625 | 466.100496342962 | 75059 | 5144.986463739997 |
| 170 | 2.23046875 | 486.692971907953 | 78309 | 5400.618548270652 |

Table 6 ($y' = x^3y$, nine nodes, 5 times) 6/7

| S=step | | E=(error of y) $\times 10^{12}$ | | |
|--------|------------|-------------------------------------|--------|--------------------|
| S | x | y | E | y' |
| 171 | 2.234375 | 508.310978019343 | 81881 | 5670.190887836494 |
| 172 | 2.23828125 | 531.010557074125 | 85461 | 5954.525032992793 |
| 173 | 2.2421875 | 554.851060689668 | 89318 | 6254.494311404638 |
| 174 | 2.24609375 | 579.895358643709 | 93381 | 6571.027290651581 |
| 175 | 2.25 | 606.210061813603 | 97490 | 6905.111485345569 |
| 176 | 2.25390625 | 633.865760103267 | 102049 | 7257.797326648693 |
| 177 | 2.2578125 | 662.937276418089 | 106746 | 7630.202413660863 |
| 178 | 2.26171875 | 693.503937833695 | 111592 | 8023.516067736979 |
| 179 | 2.265625 | 725.649865182074 | 116874 | 8439.004212343982 |
| 180 | 2.26953125 | 759.464282374071 | 122231 | 8878.014602879118 |
| 181 | 2.2734375 | 795.041846879199 | 127978 | 9341.982432819860 |
| 182 | 2.27734375 | 832.483002882905 | 134050 | 9832.436344557038 |
| 183 | 2.28125 | 871.894358764931 | 140247 | 10351.004875599892 |
| 184 | 2.28515625 | 913.389090662453 | 147049 | 10899.423373216114 |
| 185 | 2.2890625 | 957.087374013704 | 154099 | 11479.541413168349 |
| 186 | 2.29296875 | 1003.116845130707 | 161421 | 12093.330761145207 |
| 187 | 2.296875 | 1051.613094997089 | 169356 | 12742.893918454892 |
| 188 | 2.30078125 | 1102.720197660269 | 177482 | 13430.473296933254 |
| 189 | 2.3046875 | 1156.591275773187 | 186173 | 14158.461071671327 |
| 190 | 2.30859375 | 1213.389106028944 | 195378 | 14929.409763971051 |
| 191 | 2.3125 | 1273.286767454497 | 204848 | 15746.043611297032 |
| 192 | 2.31640625 | 1336.468335755083 | 215159 | 16611.270785532232 |
| 193 | 2.3203125 | 1403.129627147221 | 225904 | 17528.196525795779 |
| 194 | 2.32421875 | 1473.478995397215 | 237124 | 18500.137257610078 |
| 195 | 2.328125 | 1547.738186062738 | 249233 | 19530.635775940122 |
| 196 | 2.33203125 | 1626.143252255404 | 261733 | 20623.477576057821 |
| 197 | 2.3359375 | 1708.945536587583 | 275081 | 21782.708423148860 |
| 198 | 2.33984375 | 1796.412724325239 | 289244 | 23012.653258947818 |
| 199 | 2.34375 | 1888.829973179993 | 303919 | 24317.936551980882 |
| 200 | 2.34765625 | 1986.501125601107 | 319804 | 25703.504206777545 |
| 201 | 2.3515625 | 2089.750009893464 | 336438 | 27174.647156979404 |
| 202 | 2.35546875 | 2198.921837007657 | 353885 | 28737.026777875894 |
| 203 | 2.359375 | 2314.384700386418 | 372660 | 30396.702265091398 |
| 204 | 2.36328125 | 2436.531186854647 | 392178 | 32160.160138572789 |
| 205 | 2.3671875 | 2565.780107192876 | 413002 | 34034.346044523740 |
| 206 | 2.37109375 | 2702.578355723206 | 431132 | 36026.699042369772 |
| 207 | 2.375 | 2847.402909012176 | 458206 | 38145.188579911159 |
| 208 | 2.37890625 | 3000.762974615232 | 483083 | 40398.354377034756 |
| 209 | 2.3828125 | 3163.202301679539 | 509246 | 42795.349457116334 |
| 210 | 2.38671875 | 3335.301666210034 | 536793 | 45345.986585936589 |
| 211 | 2.390625 | 3517.681544845735 | 566382 | 48060.788400054752 |

the first value of $y_{207.5}$ for $x_{207.5}=2.376953125$
2923.0019224503 by 5^{-4}_{-1} 2923.0115658739 by 6^{-4}_{-2}
2923.0187448969 by 7^{-4}_{-3} 2923.0247263760 by 8^{-4}_{-4}
2923.0299601418 by 9^{-4}_{-5} 2923.0346705300 by 10^{-4}_{-6}
2923.0389883858 by 11^{-4}_{-7} 2923.0467566711 by 13^{-4}_{-9}

the first value of $y_{208.5}$ for $x_{208.5}=2.380859375$
3080.8096280730 by 5^{-3}_{-2} 3080.8054951772 by 6^{-3}_{-3}
3080.8003673036 by 7^{-3}_{-4} 3080.7943858245 by 8^{-3}_{-5}
3080.7876566971 by 9^{-3}_{-6} 3080.7802546585 by 10^{-3}_{-7}
3080.7722357834 by 11^{-3}_{-8} 3080.7545155013 by 13^{-3}_{-10}

Table 6 ($y' = x^3y$, nine nodes, 5 times) 7/7

$S=\text{step}$ $E=(\text{erroe of } y) \times 10^{12}$

the first value of $y_{209.5}$ for $x_{209.5}=2.384765625$
 3248.0082705802 by 5^{+2}_{-3} 3284.0124034760 by 6^{+2}_{-4}
 3248.0195824990 by 7^{+2}_{-5} 3284.0303491614 by 8^{+2}_{-6}
 3248.0451532417 by 9^{+2}_{-7} 3284.0643985421 by 10^{+2}_{-8}
 3248.0884551674 by 11^{+2}_{-9} 3284.1523555784 by 13^{+2}_{-11}
 the first value of $y_{210.5}$ for $x_{210.5}=2.388671875$
 3425.1634762938 by 5^{+1}_{-4} 3425.1538328752 by 6^{+1}_{-5}
 3425.1322958011 by 7^{+1}_{-6} 3425.0928180395 by 8^{+1}_{-7}
 3425.0286670243 by 9^{+1}_{-8} 3424.9324405223 by 10^{+1}_{-9}
 3424.7961196456 by 11^{+1}_{-10} 3424.3682911797 by 13^{+1}_{-12}

| S | x | y | E | y' |
|-----|-------------|--------------------|---------|---------------------|
| 212 | 2.392578125 | 3612.932135196011 | 581638 | 49483.246725713306 |
| 213 | 2.39453125 | 3711.004992148743 | 597429 | 50951.041530995152 |
| 214 | 2.396484375 | 3811.990158382369 | 613687 | 52465.712607893348 |
| 215 | 2.3984375 | 3915.980737661625 | 630433 | 54028.855058178064 |
| 216 | 2.400390625 | 4023.073004891346 | 647675 | 55642.121382520793 |
| 217 | 2.40234375 | 4133.366520330664 | 665270 | 57307.223652215010 |
| 218 | 2.404296875 | 4246.964248133190 | 683710 | 59025.935766912298 |
| 219 | 2.40625 | 4363.972679381413 | 702628 | 60800.095801880933 |
| 220 | 2.408203125 | 4484.501959798718 | 721942 | 62631.608448559148 |
| 221 | 2.41015625 | 4608.666022321222 | 741946 | 64522.447552214912 |
| 222 | 2.412109375 | 4736.582724723228 | 762530 | 66474.658750744475 |
| 223 | 2.4140625 | 4868.373992500127 | 783755 | 68490.362218843716 |
| 224 | 2.416015625 | 5004.165967216643 | 845620 | 70571.755521907182 |
| 225 | 2.41796875 | 5144.089160540115 | 827981 | 72721.116584242525 |
| 226 | 2.419921875 | 5288.278614187463 | 851347 | 74940.806776383513 |
| 227 | 2.421875 | 5436.874066020056 | 875354 | 77233.274126435847 |
| 228 | 2.423828125 | 5590.020122538761 | 899909 | 79601.056660722688 |
| 229 | 2.42578125 | 5747.866438033451 | 925349 | 82046.785879100428 |
| 230 | 2.427734375 | 5910.567900655065 | 951523 | 84573.190370601674 |
| 231 | 2.4296875 | 6078.284825693147 | 978533 | 87183.099575350096 |
| 232 | 2.431640625 | 6251.183156347517 | 1006376 | 89879.447698871510 |
| 233 | 2.43359375 | 6429.434622299452 | 1034901 | 92665.277785254802 |
| 234 | 2.435546875 | 6613.217206399724 | 1064644 | 95543.745955886819 |
| 235 | 2.4375 | 6802.714869800313 | 1095240 | 98518.125820723825 |
| 236 | 2.439453125 | 6998.118285879149 | 1126586 | 101591.813069493389 |
| 237 | 2.44140625 | 7199.624833312938 | 1159069 | 104768.330250414697 |
| 238 | 2.443359375 | 7407.438898671728 | 1192496 | 108051.331744410476 |
| 239 | 2.4453125 | 7621.772138928422 | 1227011 | 111444.608943188855 |
| 240 | 2.447265625 | 7842.843754286429 | 1262617 | 114952.095639845900 |
| 241 | 2.44921875 | 8070.880771751216 | 1299147 | 118577.873641100556 |
| 242 | 2.451171875 | 8306.118339888319 | 1337178 | 122326.178610657629 |
| 243 | 2.453125 | 8548.800035225272 | 1376340 | 126201.406153556946 |
| 244 | 2.455078125 | 8799.178180783736 | 1416525 | 130208.118151940666 |
| 245 | 2.45703125 | 9057.514177239463 | 1458171 | 134351.049362995487 |
| 246 | 2.458984375 | 9324.078847232552 | 1501046 | 138635.114290356846 |
| 247 | 2.4609375 | 9599.152793377393 | 1545339 | 143065.414340832050 |
| 248 | 2.462890625 | 9883.026770537452 | 1591064 | 147647.245278712817 |
| 249 | 2.46484375 | 10176.002072960939 | 1638038 | 152386.104990593099 |
| 250 | 2.466796875 | 10478.390936897274 | 1686885 | 157287.701574160260 |