

On a New Multistep Method III

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Abstract

In [2], we used the fourth order Bessel central difference formula to cut the step size in half, for the first approximate value of middle value. However, the corrections by correctors have big errors. For this reason, we did not write numerical example for initial value problems of differential equations. But fortunately we have obtained the ten-node interpolation using Taylor expansion, and the twelve-node interpolation using Taylor expansion in [3] and [4], respectively. So we shall show numerical examples for initial value problems of differential equations.

1. Introduction

In odd-node cases, the biggest error term of the corrector for y_{n+k} is higher one order than other correctors, and this fact is shown in [2]. So, we try to obtain numerical examples of initial value problems of differential equations in three cases, i.e., five-node, seven-node and nine-node. Tables 1, 3 and 5 are numerical examples $y' = x^3y$, with the starting step size $h = 0.0625$, and the initial value $y(0) = 1$. Tables 2, 4 and 6 are numerical examples for $y' = 1/x$ with the starting step size $h = 0.0625$, and initial value $y(0) = 1$. In Tables 3, 5, 8, and 9, decimal values of y and y' are written in 12 place, and in other tables, decimal values of y and y' are written in 10 place. In [1], we used $y' = x^3/y^3$. However, in our case, we need use the corrector several times for this differential equation. So, we select $y' = 1/x$ in place of $y' = x^3/y^3$.

2. Table 1 : five nodes for $y' = x^3y$, at most four times correction

We solve numerically the differential equation : $y' = x^3y$, by the following multistep method.

The predictor : $y_{n+3} = y_n + h(27y'_{n-2} - 138y'_{n-1} + 312y'_n - 198y'_{n+1} + 237y'_{n+2})/80 + 51h^6y^{(6)}/160$. (2.1)

(The correspondence of y_n between (2.1) and (2.2))

(2.1)	y_{n-2}	y_{n-1}	y_n	y_{n+1}	y_{n+2}	y_{n+3}
(2.2)	y_n	y_{n+1}	y_{n+2}	y_{n+3}	y_{n+4}	

The corrector : $y_{n+4} = y_n + h(14(y'_n + y'_{n+4}) + 64(y'_{n+1} + y'_{n+3}) + 24y'_{n+2})/45 + 0 \times h^6 y^{(6)} - 8h^7 y^{(7)}/945.$ (2.2)

If after 4 times correction, y_{n+4} is not corrected rightly, we halve the step size h .

And, when we halve the step size h , we use the following ten-node interpolation formula in [3] to get the first approximate values of middle values.

$$10_{-8}^{+2}: f(0) = (19305f(h/2) + 77220f(-h/2) - 715f(3h/2) - 60060f(-3h/2) + 54054f(-5h/2) - 38610f(-7h/2) + 20020f(-9h/2) - 7020f(-11h/2) + 1485f(-13h/2) - 143f(-15h/2))/65536 + 429h^{10}f^{(10)}/262144. \quad (2.3)$$

$$10_{-9}^{+1}: f(0) = (12155f(h/2) + 109395f(-h/2) - 145860f(-3h/2) + 204204f(-5h/2) - 218790f(-7h/2) + 170170f(-9h/2) - 92820f(-11h/2) + 33660f(-13h/2) - 7293f(-15h/2) + 715f(-17h/2))/65536 - 2431h^{10}f^{(10)}/262144. \quad (2.4)$$

In the next step, we correct these first approximate values which are gotten from an interpolation formula by the following correctors :

$$y_{n-1} = y_n - h(-19y'_{n-2} + 346y'_{n-1} + 456y'_n - 74y'_{n+1} + 11y'_{n+2})/720 + 11h^6 y^{(6)}/1440. \quad (2.5)$$

$$y_{n+1} = y_n + h(11y'_{n-2} - 74y'_{n-1} + 456y'_n + 346y'_{n+1} - 19y'_{n+2})/720 + 11h^6 y^{(6)}/1440. \quad (2.6)$$

First, as starting values the five values of y which are gotten by the Taylor expansion is used, and the starting step size h is 0.0625, and $y(0) = 1$.

In this table, decimal values of y and y' are written at 10 places.

3. Table 2 : five nodes for $y' = 1/x$, at most four times correction

By the numerical method, we solve the differential equation : $y' = 1/x$, by predictor (2.1) and corrector (2.2) in Section 2.

First, as starting values, the five values gotten by the Taylor expansion is used, and the starting step size h is 0.0625, and $y(0) = 1$. In this table, decimal values of y and y' are written at 10 places.

We introduce the method of doubling the step size, when one or two times corrections of the value of y are consecutive in twenty steps. It is noticed that we stop doubling the step size when $h > 0.5$.

4. Table 3 : seven nodes for $y' = x^3 y$, at most four times correction

We solve numerically the differential equation : $y' = x^3 y$, by the following method.

The predictor : $y_{n+4} = y_n + h(286y'_{n-3} - 2010y'_{n-2} + 6054y'_{n-1} - 9836y'_n + 11514y'_{n+1} - 5622y'_{n+2} + 3394y'_{n+3})/945 + 278h^8 y^{(8)}/945.$ (4.1)

(The correspondence of y_n between (4.1) and (4.2))

(4.1)	y_{n-3}	y_{n-2}	y_{n-1}	y_n	y_{n+1}	y_{n+2}	y_{n+3}	y_{n+4}
(4.2)	y_n	y_{n+1}	y_{n+2}	y_{n+3}	y_{n+4}	y_{n+5}	y_{n+6}	

The corrector : $y_{n+6} = y_n + h(41(y'_n + y'_{n+6}) + 216(y'_{n+1} + y'_{n+5}) + 27(y'_{n+2} + y'_{n+4}) + 272y'_{n+3})/140 + 0 \times h^8 y^{(8)} - 9h^9 y^{(9)}/1400.$ (4.2)

If after four times correction we can't correct y_{n+6} , we halve the step size h . When we halve the step size h , we use (2.3), (2.4) in Section 2, and the following ten-node interpolation formula in [3] to get the first approximate values of middle values.

$$10_{-7}^{+3}: f(0) = (25740f(h/2) + 60060f(-h/2) - 2145f(3h/2) - 30030f(-3h/2) + 143f(5h/2) + 18018f(-5h/2) - 8580f(-7h/2) + 2860f(-9h/2) - 585f(-11h/2) + 55f(-13h/2))/65536 - 143h^{10}f^{(10)}/262144. \quad (4.3)$$

In the next step, we correct these first approximate values by the following correctors :

$$y_{n-3} = y_n - h(685y'_{n-3} + 3240y'_{n-2} + 1161y'_{n-1} + 2176y'_n - 729y'_{n+1} + 216y'_{n+2} - 29y'_{n+3})/2240 - 9h^8 y^{(8)}/896. \quad (4.4)$$

$$y_{n-1} = y_n - h(271y'_{n-3} - 2760y'_{n-2} + 30819y'_{n-1} + 37504y'_n - 6771y'_{n+1} + 1608y'_{n+2} - 191y'_{n+3})/60480 - 191h^8 y^{(8)}/120960. \quad (4.5)$$

$$y_{n+1} = y_n + h(-191y'_{n-3} + 1608y'_{n-2} - 6771y'_{n-1} + 37504y'_n + 30819y'_{n+1} - 2760y'_{n+2} + 271y'_{n+3})/60480 - 191h^8 y^{(8)}/120960. \quad (4.6)$$

$$y_{n+3} = y_n + h(-29y'_{n-3} + 216y'_{n-2} - 729y'_{n-1} + 2176y'_n + 1161y'_{n+1} + 3240y'_{n+2} + 685y'_{n+3})/2240 - 9h^8 y^{(8)}/896. \quad (4.7)$$

Here, at the seven-node formula, the value of y_n is gotten by the 10_{-8}^{+2} formula, so we must select the pillar value for y_n . From this reason we must accept big error terms of (4.4) and (4.7). At the five-node formula and at the nine-node formula the value of y_n is the pillar value.

First as starting values, the seven values gotten by the Taylor expansion is used, and the starting step size h is 0.0625, and $y(0) = 1$.

In this table, decimal values of y and y' are written at 12 places.

5. Table 4 : seven nodes for $y' = 1/x$, at most four times correction

We solve numerically the differential equation $y' = 1/x$, by predictor (4.1) and corrector (4.2) in Section 4.

First, as starting values, the seven values gotten by the Taylor expansion is used, and the starting step size h is 0.0625, and $y(0) = 1$.

We introduce the method of doubling the step size, in the case at most two times corrections of the value of y are consecutive in twenty steps. It is noticed that we stop doubling the step size when $h > 0.5$.

In this table, decimal values of y and y' are written at 10 places.

6. Table 5 : nine nodes for $y' = x^3 y$, at most four times correction

We solve numerically the differential equation : $y' = x^3 y$, by the following multistep method.

The predictor : $y_{n+5} = y_n + h(41705y'_{n-4} - 376570y'_{n-3} + 1512830y'_{n-2} - 3552370y'_{n-1} + 5425760y'_n - 5258830y'_{n+1} + 3821570y'_{n+2} - 1493830y'_{n+3} + 605495y'_{n+4})/145152 + 81385h^{10}y^{(10)}/290304.$ (6.1)

(The correspondence of y'_n between (6.1) and (6.2))

(6.1)	y_{n-4}	y_{n-3}	y_{n-2}	y_{n-1}	y_n	y_{n+1}	y_{n+2}	y_{n+3}	y_{n+4}	y_{n+5}
(6.2)	y_n	y_{n+1}	y_{n+2}	y_{n+3}	y_{n+4}	y_{n+5}	y_{n+6}	y_{n+7}	y_{n+8}	

The corrector : $y_{n+8} = y_n + h(3956(y'_n + y'_{n+8}) + 23552(y'_{n+1} + y'_{n+7}) - 3712(y'_{n+2} + y'_{n+6}) + 41984(y'_{n+3} + y'_{n+5}) - 18160y'_{n+4})/14175 + 0 \times h^{10}y^{(10)} - 2368h^{11}y^{(11)}/467775.$ (6.2)

If we can't correct y_{n+8} rightly after 4 times correction, we halve half the step size h . And, when we halve the step size h , we use the twelve-node interpolation formula in [4] to get the first approximate values of middle values.

$$12_{-8}^{+4}: f(0) = (225225f(h/2) + 450450f(-h/2) - 25025f(3h/2) - 210210f(-3h/2) + 3003f(5h/2) + 126126f(-5h/2) - 195f(7h/2) - 64350f(-7h/2) + 25025f(-9h/2) - 6825f(-11h/2) + 1155f(-13h/2) - 91f(-15h/2))/524288 + 455h^{12}f^{(12)}/4194304. \quad (6.3)$$

$$12_{-9}^{+3}: f(0) = (182325f(h/2) + 546975f(-h/2) - 12155f(3h/2) - 364650f(-3h/2) + 663f(5h/2) + 306306f(-5h/2) - 218790f(-7h/2) + 121550f(-9h/2) - 49725f(-11h/2) + 14025f(-13h/2) - 2431f(-15h/2) + 195f(-17h/2))/524288 - 1105h^{12}f^{(12)}/4194304. \quad (6.4)$$

$$12_{-10}^{+2}: f(0) = (138567f(h/2) + 692835f(-h/2) - 4199f(3h/2) - 692835f(-3h/2) + 831402f(-5h/2) - 831402f(-7h/2) + 646646f(-9h/2) - 377910f(-11h/2) + 159885f(-13h/2) - 46189f(-15h/2) + 8151f(-17h/2) - 663f(-19h/2))/524288 - 4199h^{12}f^{(12)}/4194304. \quad (6.5)$$

$$12_{-11}^{+1}: f(0) = (88179f(h/2) + 969969f(-h/2) - 1616615f(-3h/2) + 2909907f(-5h/2) - 4157010f(-7h/2) + 4526522f(-9h/2) - 3703518f(-11h/2) + 2238390f(-13h/2) - 969969f(-15h/2) + 285285f(-17h/2) - 51051f(-19h/2) + 4199f(-21h/2))/524288 - 29393h^{12}f^{(12)}/4194304. \quad (6.6)$$

We correct these first approximate values gotten from interpolation formulas by the following correctors.

$$y_{n-3} = y_n - h(-369y'_{n-4} + 16202y'_{n-3} + 57618y'_{n-2} + 34434y'_{n-1} + 33440y'_n - 9666y'_{n+1} + 3438y'_{n+2} - 778y'_{n+3} + 81y'_{n+4})/44800 + 113h^{10}y^{(10)}/89600. \quad (6.7)$$

$$y_{n-1} = y_n - h(-3233y'_{n-4} + 36394y'_{n-3} - 216014y'_{n-2} + 1909858y'_{n-1} + 2224480y'_n - 425762y'_{n+1} + 126286y'_{n+2} - 25706y'_{n+3} + 2497y'_{n+4})/3628800 + 2497h^{10}y^{(10)}/7257600. \quad (6.8)$$

$$y_{n+1} = y_n + h(2497y'_{n-4} - 25706y'_{n-3} + 126286y'_{n-2} - 425762y'_{n-1} + 2224480y'_n + 1909858y'_{n+1} - 216014y'_{n+2} + 36394y'_{n+3} - 3233y'_{n+4})/3628800 + 2497h^{10}y^{(10)}/7257600. \quad (6.9)$$

$$y_{n+3} = y_n + h(81y'_{n-4} - 778y'_{n-3} + 3438y'_{n-2} - 9666y'_{n-1} + 33440y'_n + 34434y'_{n+1} + 57618y'_{n+2} + 16202y'_{n+3} - 369y'_{n+4})/44800 + 113h^{10}y^{(10)}/89600. \quad (6.10)$$

First nine starting values are gotten by Taylor expansion, the starting step size $h = 0.0625$, and $y = 1$ when $x = 0$ for the initial value. In this table, decimal values of y and y' are written at 12 places.

7. Table 6 : nine nodes for $y' = 1/x$, at most four times correction

We solve numerically the differential equation : $y' = 1/x$, by predictor (6.1) and corrector (6.2) in Section 6. First as starting values, the nine values gotten by the Taylor expansion is used, and the starting step size h is 0.0625, and $y(0) = 1$.

We introduce the method of doubling the step size, when one or two times corrections of the value of y are consecutive in twenty steps. It is noticed that we stop doubling the step size when $h > 0.5$. In this table, decimal values of y and y' are written at 10 places.

8. Table 7 : five nodes for $y' = x^3y$, at most five times correction

We solve numerically $y' = x^3y$ by the same method as Table 1. Except that at most five times we correct the value of y_{n+4} gotten by the predictor. (At Table 1 we correct the value of y_{n+4} at most four times.)

9. Table 8 : seven nodes for $y' = x^3y$, at most five times correction

We use the same method as Table 3. Except that at most five times we correct the value of y_{n+6} gotten by the predictor. (In Table 3 we correct the value of y_{n+6} at most four times.)

10. Table 9 : nine nodes for $y' = x^3y$, at most five times correction

We employ the same method as Table 5. Except that at most five times we correct the value of y_{n+8} gotten by the predictor. (In Table 5, we correct the value of y_{n+8} at most four times.)

11. Conclusion

In Table 1, 3, and 5, if we can not correct y after using correctors at most four times, we cut the step size h in half. Moreover, in Tables 7, 8 and 9, if we can not correct y after using correctors at most five times, we cut the step size h in half.

We show the values of x , y , and errors of y at 100 step and at 200 step, at the following Table I and Table II, respectively.

Table I (at 100 step)

nodes	Table No.	correct times	x	y	the error of y
5	1	4	1.27734375	1.9455318828	102×10^{-10}
5	7	5	1.55078125	4.2457269888	764×10^{-10}
7	3	4	1.53125	3.952845357449	98×10^{-10}
7	8	5	1.765625	11.354282770319	9690×10^{-12}
9	5	4	1.6875	7.593453766227	229×10^{-12}
9	9	5	1.8984375	24.390561636165	3949×10^{-12}

Table II (at 200 step)

nodes	Table No.	correct times	x	y	the error of y
5	1	4	1.640625	6.1180635009	321×10^{-10}
5	7	5	1.91796875	29.4599614233	6675×10^{-10}
7	3	4	1.9375	33.884342464025	842×10^{-12}
7	8	5	2.17578125	271.169997771836	231132×10^{-12}
9	5	4	2.125	163.647919079462	5419×10^{-12}
9	9	5	2.34765625	1986.501125601106	319804×10^{-12}

From above Table I and Table II, we see that in the case of many nodes, even if the program itself is complicated, but we obtain a highly precise numerical solution using by the interpolations of many nodes. At most 5 times correction method, we need more step of x than at most 4 times correction method. But, we get low accurate at 5 times correction method than 4 times correction method. This fact is natural results.

References

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Table 1 ($y' = x^3y$, five nodes, 4 times) 1/4

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
1	-0.125	1.0000610370	0	-0.0019532442
2	-0.0625	1.0000038147	0	-0.0002441416
3	0	1.0000000000	0	0.0000000000
4	0.0625	1.0000038147	0	0.0002441416
5	0.125	1.0000610370	0	0.0019532442
6	0.1875	1.0003090407	25	0.0065938340
7	0.25	1.0009770445	50	0.0156402663
8	0.3125	1.0023870378	76	0.0305904247
9	0.34375	1.0034967908	48	0.0407609323
10	0.375	1.0049560937	51	0.0529957315
11	0.40625	1.0068327151	48	0.0675052330
12	0.4375	1.0092011687	77	0.0845107424
13	0.46875	1.0121430813	50	0.1042475250
14	0.5	1.0157477139	53	0.1269684642
15	0.515625	1.0178286716	50	0.1395328864
16	0.53125	1.0201125487	51	0.1529483933
17	0.546875	1.0226129424	51	0.1672536083
18	0.5625	1.0253440697	54	0.1824892155
19	0.578125	1.0283207995	51	0.1986981715
20	0.59375	1.0315586931	51	0.2159259362
21	0.609375	1.0350740419	51	0.2342207225
22	0.625	1.0388839147	55	0.2536337682
23	0.640625	1.0430062049	52	0.2742196299
24	0.65625	1.0474596886	52	0.2960365044
25	0.671875	1.0522640803	52	0.3191465768
26	0.6875	1.0574401007	56	0.3436163999
27	0.703125	1.0630095455	53	0.3695173066
28	0.71875	1.0689953675	53	0.3969258617
29	0.734375	1.0754217585	53	0.4259243516
30	0.75	1.0823142447	57	0.4566013220
31	0.765625	1.0896997874	54	0.4890521633
32	0.78125	1.0976068978	55	0.5233797540
33	0.796875	1.1060657564	55	0.5596951624
34	0.8125	1.1151083487	59	0.5981184185
35	0.828125	1.1247686097	56	0.6387793591
36	0.8359375	1.1298415445	59	0.6599924446
37	0.84375	1.1350825884	60	0.6818185604
38	0.8515625	1.1404966064	60	0.7042771243
39	0.859375	1.1460886177	57	0.7273883582
40	0.8671875	1.1518638039	61	0.7511733275
41	0.875	1.1578275129	61	0.7756539783
42	0.8828125	1.1639852685	61	0.8008531818
43	0.890625	1.1703427767	59	0.8267947763
44	0.8984375	1.1769059348	62	0.8535036152
45	0.90625	1.1836808364	62	0.8810056128
46	0.9140625	1.1906737833	63	0.9093277979
47	0.921875	1.1978912922	60	0.9384983662
48	0.9296875	1.2053401061	63	0.9685467376
49	0.9375	1.2130272011	64	0.9995036142
50	0.9453125	1.2209597993	64	1.0314010444

Table 1 ($y' = x^3y$, five nodes, 4 times) 2/4

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
51	0.953125	1.2291453784	62	1.0642724882
52	0.9609375	1.2375916843	65	1.0981528877
53	0.96875	1.2463067400	66	1.1330787382
54	0.9765625	1.2552988613	66	1.1690881674
55	0.984375	1.2645766677	63	1.2062210161
56	0.9921875	1.2741490985	67	1.2445189251
57	1.0	1.2840254234	68	1.2840254234
58	1.0078125	1.2942152614	68	1.3247860267
59	1.015625	1.3047285945	66	1.3668483362
60	1.0234375	1.3155757860	69	1.4102621481
61	1.03125	1.3267675959	70	1.4550795621
62	1.0390625	1.3383152012	70	1.5013551044
63	1.046875	1.3502302147	68	1.5491458513
64	1.0546875	1.3625247063	72	1.5985115644
65	1.0625	1.3752112224	72	1.6495148280
66	1.0703125	1.3883028104	73	1.7022212012
67	1.078125	1.4018130421	71	1.7566993738
68	1.0859375	1.4157560394	74	1.8130213350
69	1.09375	1.4301464985	75	1.8712625465
70	1.1015625	1.4449997204	76	1.9315021331
71	1.109375	1.4603316389	74	1.9938230789
72	1.1171875	1.4761588526	77	2.0583124399
73	1.125	1.4924986555	78	2.1250615622
74	1.1328125	1.5093690736	79	2.1941663227
75	1.140625	1.5267889001	77	2.2657273771
76	1.1484375	1.5447777352	81	2.3398504285
77	1.15625	1.5633560239	82	2.4166465050
78	1.1640625	1.5825451012	83	2.4962322640
79	1.171875	1.6023672362	81	2.5787303077
80	1.1796875	1.6228456814	85	2.6642695243
81	1.1875	1.6440047199	86	2.7529854428
82	1.1953125	1.6658697222	87	2.8450206193
83	1.203125	1.6884672006	86	2.9405250415
84	1.2109375	1.7118248708	90	3.0396565631
85	1.21875	1.7359717122	91	3.1425813598
86	1.2265625	1.7483504910	92	3.1955205628
87	1.2265625	1.7609380377	92	3.2494744233
88	1.23046875	1.7737383614	93	3.3044662778
89	1.234375	1.7867555633	94	3.3605200812
90	1.23828125	1.7999938393	94	3.4176604242
91	1.2421875	1.8134574829	95	3.4759125526
92	1.24609375	1.8271508875	96	3.5353023866
93	1.25	1.8410785489	97	3.5958565408
94	1.25390625	1.8552450683	97	3.6576023444
95	1.2578125	1.8696551553	98	3.7205678635
96	1.26171875	1.8843136304	99	3.7847819226
97	1.265625	1.8992254283	100	3.8502741274
98	1.26953125	1.9143956008	100	3.9170748880
99	1.2734375	1.9298293203	101	3.9852154443
100	1.27734375	1.9455318828	102	4.0547278900

Table 1 ($y' = x^3y$, five nodes, 4 times) 3/4

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
101	1.28125	1.9615087112	103	4.1256451991
102	1.28515625	1.9777653592	104	4.1980012525
103	1.2890625	1.9943075146	104	4.2718308659
104	1.29296875	2.0111410031	105	4.3471698186
105	1.296875	2.0282717920	107	4.4240548825
106	1.30078125	2.0457059942	107	4.5025238533
107	1.3046875	2.0634498725	108	4.5826155822
108	1.30859375	2.0815098434	109	4.6643700079
109	1.3125	2.0998924816	110	4.7478281914
110	1.31640625	2.1186045244	111	4.8330323499
111	1.3203125	2.1376528767	112	4.9200258941
112	1.32421875	2.1570446147	113	5.0088534650
113	1.328125	2.1767869922	114	5.0995609724
114	1.33203125	2.1968874441	115	5.1921956355
115	1.3359375	2.2173535930	116	5.2868060240
116	1.33984375	2.2381932537	117	5.3834421009
117	1.34375	2.2594144390	119	5.4821552673
118	1.34765625	2.2810253651	120	5.5829984073
119	1.3515625	2.3030344582	121	5.6860259369
120	1.35546875	2.3254503598	122	5.7912938520
121	1.359375	2.3482819335	123	5.8988597797
122	1.36328125	2.3715382712	124	6.0087830309
123	1.3671875	2.3952286998	126	6.1211246552
124	1.37109375	2.4193627882	127	6.2359474969
125	1.375	2.4439503546	128	6.3533162537
126	1.37890625	2.4690014731	129	6.4732975370
127	1.3828125	2.4945264825	131	6.5959599352
128	1.38671875	2.5205359930	132	6.7213740781
129	1.390625	2.5470408951	134	6.8496127044
130	1.39453125	2.5740523674	135	6.9807507314
131	1.3984375	2.6015818858	136	7.1148653275
132	1.40234375	2.6296412319	138	7.2520359868
133	1.40625	2.6582425028	140	7.3923446065
134	1.41015625	2.6873981198	141	7.5358755675
135	1.4140625	2.7171208395	142	7.6827158183
136	1.41796875	2.7474237627	144	7.8329549603
137	1.421875	2.7783203459	146	7.9866853385
138	1.42578125	2.8098244116	147	8.1440021330
139	1.4296875	2.8419501599	149	8.3050034570
140	1.43359375	2.8747121802	151	8.4697904554
141	1.4375	2.9081254628	153	8.6384674087
142	1.44140625	2.9422054117	154	8.8111418405
143	1.4453125	2.9769678577	156	8.9879246292
144	1.44921875	3.0124290711	158	9.1689301231
145	1.453125	3.0486057760	160	9.3542762611
146	1.45703125	3.0855151645	162	9.5440846959
147	1.4609375	3.1231749112	164	9.7384809250
148	1.46484375	3.1616031888	166	9.9375944232
149	1.46875	3.2008186837	168	10.1415587829
150	1.47265625	3.2408406123	170	10.3505118574

Table 1 ($y' = x^3y$, five nodes, 4 times) 4/4

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
151	1.4765625	3.2816887383	172	10.5645959123
152	1.48046875	3.3233833898	174	10.7839577804
153	1.484375	3.3659454778	177	11.0087490235
154	1.48828125	3.4093965146	179	11.2391261005
155	1.4921875	3.4537586337	181	11.4752505422
156	1.49609375	3.4990546099	183	11.7172891320
157	1.5	3.5453078798	186	11.9654140944
158	1.50390625	3.5925425638	188	12.2198032908
159	1.5078125	3.6407834885	191	12.4806404226
160	1.51171875	3.6900562097	193	12.7481152426
161	1.515625	3.7403870366	196	13.0224237743
162	1.51953125	3.7918030569	199	13.3037685403
163	1.5234375	3.8443321623	202	13.5923587998
164	1.52734375	3.8980030755	204	13.8884107948
165	1.53125	3.9528453781	208	14.1921480068
166	1.53515625	4.0088895390	210	14.5038014235
167	1.5390625	4.0661669448	213	14.8236098162
168	1.54296875	4.1247099302	216	15.1518200285
169	1.546875	4.1845518106	220	15.4886872760
170	1.55078125	4.2457269150	223	15.8344754587
171	1.5546875	4.3082706208	226	16.1894574863
172	1.55859375	4.3722193898	229	16.5539156159
173	1.5625	4.4376108050	233	16.9281418038
174	1.56640625	4.5044836093	236	17.3124380716
175	1.5703125	4.5728777459	240	17.7071168881
176	1.57421875	4.6428343994	243	18.1125015656
177	1.578125	4.7143960395	247	18.5289266733
178	1.58203125	4.7876064650	251	18.9567384671
179	1.5859375	4.8625108512	255	19.3962953389
180	1.58984375	4.9391557979	259	19.8479682824
181	1.59375	5.0175893791	263	20.3121413797
182	1.59765625	5.0978611958	267	20.7892123072
183	1.6015625	5.1800224298	272	21.2795928649
184	1.60546875	5.2641259000	276	21.7837095250
185	1.609375	5.3502261206	281	22.3020040056
186	1.61328125	5.4383793620	285	22.8349338677
187	1.615234375	5.4832439459	288	23.1070345399
188	1.6171875	5.5286437144	290	23.3829731393
189	1.619140625	5.5745862237	293	23.6628115988
190	1.62109375	5.6210791521	295	23.9466129628
191	1.623046875	5.6681303023	297	24.2344414100
192	1.625	5.7157476032	300	24.5263622740
193	1.626953125	5.7639391126	303	24.8224420679
194	1.62890625	5.8127130192	305	25.1227485062
195	1.630859375	5.8620776455	308	25.4273505296
196	1.6328125	5.9120414495	310	25.7363183281
197	1.634765625	5.9626130278	313	26.0497233668
198	1.63671875	6.0138011176	315	26.3676384105
199	1.638671875	6.0656145997	318	26.6901375504
200	1.640625	6.1180625009	321	27.0172962290
205	1.650390625	6.3901447921	335	28.7257213089

Table 2 ($y' = 1/x$, five nodes, 5 times) 1/2

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
1	0.875	-0.1335313926	0	1.1428571429
2	0.9375	-0.0645385211	0	1.0666666667
3	1	0.0000000000	0	1.0000000000
4	1.0625	0.0606246218	0	0.9411764706
5	1.125	0.1177830357	0	0.8888888889
6	1.1875	0.1718502722	153	0.8421052632
7	1.25	0.2231435615	102	0.8000000000
8	1.3125	0.2719337225	70	0.7619047619
9	1.375	0.3184537360	49	0.7272727273
10	1.4375	0.3629055124	187	0.6956521739
11	1.5	0.4054651208	127	0.6666666667
12	1.5625	0.4462871114	88	0.6400000000
13	1.625	0.4855078220	62	0.6153846154
14	1.6875	0.5232481635	197	0.5925925926
15	1.75	0.5596158014	135	0.5714285714
16	1.8125	0.5947071171	94	0.5517241379
17	1.875	0.6286086661	67	0.5333333333
18	1.9375	0.6613985023	201	0.5161290323
19	2.0	0.6931471943	137	0.5000000000
20	2.0625	0.7239188488	96	0.4848484848
21	2.125	0.7537718092	68	0.4705882353
22	2.1875	0.7827593595	202	0.4571428571
23	2.25	0.8109302301	139	0.4444444444
24	2.3125	0.8383292001	97	0.4324324324
25	2.375	0.8649974444	69	0.4210526316
26	2.5	0.9162907558	239	0.4000000000
27	2.625	0.9650809099	138	0.3809523810
28	2.75	1.0116009304	187	0.3636363636
29	2.875	1.0560526846	104	0.3478260870
30	3.0	1.0986123151	264	0.3333333333
31	3.125	1.1394342988	156	0.3200000000
32	3.25	1.1786550164	201	0.3076923077
33	3.375	1.2163953357	114	0.2962962963
34	3.5	1.2527629957	272	0.2857142857
35	3.625	1.2878543045	162	0.2758620690
36	3.75	1.3217558605	205	0.2666666667
37	3.875	1.3545456745	117	0.2580645161
38	4.0	1.3862943886	275	0.2500000000
39	4.125	1.4170660362	164	0.2424242424
40	4.25	1.4469190036	207	0.2352941176
41	4.375	1.4759065317	119	0.2285714286
42	4.5	1.5040774244	276	0.2222222222
43	4.625	1.5314763875	165	0.2162162162
44	4.75	1.5581446388	208	0.2105263158
45	4.875	1.5841201164	119	0.2051282051
46	5.125	1.6341305499	249	0.1951219512
47	5.375	1.6817585917	177	0.1860465116
48	5.625	1.7272209687	206	0.1777777778
49	5.875	1.7707060749	149	0.1702127660
50	6.125	1.8123787834	270	0.1632653061

Table 2 ($y' = 1/x$, five nodes, 5 times) 2/2

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
51	6.375	1.8523841103	192	0.1568627451
52	6.625	1.8908503937	218	0.1509433962
53	6.875	1.9278916593	157	0.1454545455
54	7.125	1.9636097538	276	0.1403508772
55	7.375	1.9980959220	198	0.1355932203
56	7.625	2.0314323447	222	0.1311475410
57	7.875	2.0636932008	161	0.1269841270
58	8.125	2.0949457561	279	0.1230769231
59	8.375	2.1252510977	200	0.1194029851
60	8.625	2.1546649853	223	0.1159420290
61	8.875	2.1832383515	162	0.1126760563
62	9.125	2.2110179275	280	0.1095890411
63	9.375	2.2380465919	200	0.1066666667
64	9.625	2.2643639026	224	0.1038961039
65	9.875	2.2900063270	162	0.1012658228
66	10.375	2.3393990937	276	0.0963855422
67	10.875	2.3864665985	215	0.0919540230
68	11.375	2.4314179886	238	0.0879120879
69	11.875	2.4744353689	189	0.0842105263
70	12.375	2.5156783380	296	0.0808080808
71	12.875	2.5552874695	229	0.0776699029
72	13.375	2.5933873177	249	0.0747663551
73	13.875	2.6300886794	198	0.0720720721
74	14.375	2.6654906169	302	0.0695652174
75	14.875	2.6996819749	234	0.0672268908
76	15.375	2.7327428390	253	0.0650406504
77	15.875	2.7647455648	201	0.0629921260
78	16.375	2.7957558120	304	0.0610687023
79	16.875	2.8258332604	236	0.0592592593
80	17.375	2.8550324169	254	0.0575539568
81	17.875	2.8834031088	202	0.0559440559
82	18.375	2.9109910756	305	0.0544217687
83	18.875	2.9378383188	237	0.0529801325
84	19.375	2.9639836007	255	0.0516129032
85	19.875	2.9894626808	202	0.0503144654
86	20.375	3.0143086897	306	0.0490797546
87	20.875	3.0385522945	237	0.0479041916
88	21.375	3.0622220403	255	0.0467836257
89	21.875	3.0853444525	203	0.0457142857
90	22.375	3.1079442948	306	0.0446927374
91	22.875	3.1300446349	238	0.0437158470
92	23.375	3.1516671007	255	0.0427807487
93	23.875	3.1728319066	203	0.0418848168
94	24.375	3.1935580475	306	0.0410256410
95	24.875	3.2138633068	238	0.0402010050
96	25.375	3.2337644629	255	0.0394088670
97	25.875	3.2532772719	203	0.0386473430
98	26.375	3.2724166224	306	0.0379146919
99	26.875	3.2911965102	238	0.0372093023
100	27.375	3.3096302137	255	0.0365296804
105	29.875	3.3970220305	203	0.0334728033

Table 3 ($y' = x^3y$, seven nodes, 4 times) 1/4S= step E=(error of y) $\times 10^{12}$

S	x	y	E	y'
1	-0.1875	1.000309038221	0	-0.006593833992
2	-0.125	1.000061037019	0	-0.001953244213
3	-0.0625	1.000003814705	0	-0.000244141556
4	0	1.000000000000	0	0.000000000000
5	0.0625	1.000003814705	0	0.000244141556
6	0.125	1.000061037019	0	0.001953244213
7	0.1875	1.000309038221	0	0.006593833992
8	0.25	1.000977039504	12	0.015640266242
9	0.3125	1.002387030274	52	0.030590424508
10	0.375	1.004956088785	150	0.052995731245
11	0.40625	1.006832710229	15	0.067505232677
12	0.4375	1.009201160947	13	0.084510741749
13	0.46875	1.012143076268	16	0.104247524487
14	0.5	1.015747708642	55	0.126968463580
15	0.53125	1.020112543704	30	0.152948392554
16	0.5625	1.025344064463	155	0.182489214598
17	0.59375	1.031558687975	23	0.215925935084
18	0.625	1.038883909208	22	0.253633766896
19	0.65625	1.047459683438	28	0.296036502940
20	0.6875	1.057440095159	68	0.343616398110
21	0.703125	1.063009540236	23	0.369517304817
22	0.71875	1.068995362231	22	0.396925859749
23	0.734375	1.075421753178	23	0.425924349519
24	0.75	1.082314239057	29	0.456601319602
25	0.765625	1.089699782030	24	0.489052160858
26	0.78125	1.097606892439	69	0.523379751415
27	0.796875	1.106065750924	24	0.559695159629
28	0.8125	1.115108342803	23	0.598118415317
29	0.828125	1.124768604150	25	0.638779355927
30	0.84375	1.135082582414	30	0.681818556813
31	0.859375	1.146088612007	25	0.727388354579
32	0.875	1.157827506823	71	0.775653974298
33	0.890625	1.170342770902	27	0.826794772231
34	0.90625	1.183680830239	25	0.881005608176
35	0.921875	1.197891286235	28	0.938498361495
36	0.9375	1.213027194718	32	0.999503608929
37	0.953125	1.229145372314	28	1.064272482884
38	0.96875	1.246306733527	73	1.133078732254
39	0.984375	1.264576661419	30	1.206221010047
40	1.0	1.284025416716	28	1.284025416716
41	1.015625	1.304728587948	32	1.366848329411
42	1.03125	1.326767588905	36	1.455079554519
43	1.046875	1.350230207899	33	1.549145843575
44	1.0625	1.375211215203	77	1.649514819408
45	1.078125	1.401813035066	35	1.756699364992
46	1.09375	1.430146491005	33	1.871262536677
47	1.109375	1.460331631535	38	1.993823068902
48	1.125	1.492498647645	41	2.125061551042
49	1.1328125	1.509369065677	35	2.194166311192
50	1.140625	1.526788892413	35	2.265727365721

Table 3 ($y' = x^3y$, seven nodes, 4 times) 2/4S= step E=(error of y) $\times 10^{12}$

S	x	y	E	y'
51	1.1484375	1.544777727169	36	2.339850416297
52	1.15625	1.563356015688	41	2.416646492390
53	1.1640625	1.582545092894	36	2.496232250926
54	1.171875	1.602367228142	44	2.578730294694
55	1.1796875	1.622845672891	38	2.664269510425
56	1.1875	1.644004711287	38	2.752985428398
57	1.1953125	1.665869713466	39	2.845020604490
58	1.203125	1.688467192050	44	2.940525026658
59	1.2109375	1.711824861854	40	3.039656547279
60	1.21875	1.735971703160	47	3.142581343376
61	1.2265625	1.760938028563	41	3.249474406323
62	1.234375	1.786755553941	41	3.360520063628
63	1.2421875	1.813457473474	42	3.475912534484
64	1.25	1.841078539241	47	3.595856521955
65	1.2578125	1.869655145541	44	3.720567844124
66	1.265625	1.899225418381	51	3.850274107246
67	1.2734375	1.929829310267	45	3.985215423560
68	1.28125	1.961508700951	45	4.125645177559
69	1.2890625	1.994307504222	47	4.271830843641
70	1.296875	2.028271781378	52	4.424054859385
71	1.3046875	2.063449861713	48	4.582615558283
72	1.3125	2.099892470601	56	4.747828166561
73	1.3203125	2.137652865507	50	4.920025868466
74	1.328125	2.176786980774	50	5.099560945769
75	1.3359375	2.217353581464	52	5.286805996383
76	1.34375	2.259414427141	57	5.482155238608
77	1.3515625	2.303034446155	54	5.686025907252
78	1.359375	2.348281921275	62	5.898859748861
79	1.3671875	2.395228687298	57	6.121124623294
80	1.375	2.443950341773	57	6.353316220507
81	1.3828125	2.494526469465	59	6.595959900784
82	1.390625	2.547040881797	64	6.849612668609
83	1.3984375	2.601581872238	62	7.114865290423
84	1.40625	2.658242488874	69	7.392344567828
85	1.4140625	2.717120825287	65	7.682715778189
86	1.421875	2.778320331351	65	7.986685296694
87	1.4296875	2.841950145118	68	8.305003413678
88	1.4375	2.908125447613	73	8.638467363551
89	1.4453125	2.976967842151	71	8.987924582272
90	1.453125	3.048605760089	79	9.354276212189
91	1.4609375	3.123174894899	75	9.738480874181
92	1.46875	3.200818666964	75	10.141558729866
93	1.4765625	3.281688721196	79	10.564595857172
94	1.484375	3.365945460205	84	11.008748965999
95	1.4921875	3.453758615723	83	11.475250482319
96	1.5	3.545307861314	91	11.965414031936
97	1.5078125	3.640783469507	88	12.480640357466
98	1.515625	3.740387017097	88	13.022423706265
99	1.5234375	3.844332142211	93	13.592358728876
100	1.53125	3.952845357449	98	14.192147932696

Table 3 ($y' = x^3y$, seven nodes, 4 times) 3/4

S	x	y	E	y'
101	1.5390625	4.066166923584	98	14.823609738856
102	1.546875	4.184551788748	107	15.488687195095
103	1.5546875	4.308270598365	105	16.189457401851
104	1.5625	4.437610781826	105	16.928141715340
105	1.56640625	4.504483585810	114	17.312437981238
106	1.5703125	4.572877722035	116	17.707116795748
107	1.57421875	4.642834375212	118	18.112501471139
108	1.578125	4.714396014826	115	18.528926576505
109	1.58203125	4.787606439978	122	18.956738368188
110	1.5859375	4.862510825848	116	19.396295237704
111	1.58984375	4.939155772090	125	19.847968178856
112	1.59375	5.017589352905	128	20.312141273566
113	1.59765625	5.097861169227	129	20.789212198735
114	1.6015625	5.180022402813	127	21.279592753904
115	1.60546875	5.264125872505	133	21.783709411345
116	1.609375	5.305226092603	128	22.302003889051
117	1.61328125	5.438379333660	137	22.834933748517
118	1.6171875	5.528643685488	141	23.382973017144
119	1.62109375	5.621079122803	142	23.946612837839
120	1.625	5.715747573371	140	24.526362145889
121	1.62890625	5.812712988891	147	25.122748375050
122	1.6328125	5.912041418645	142	25.736318193645
123	1.63671875	6.013801086195	152	26.367638272926
124	1.640625	6.118062468982	155	27.017296087855
125	1.64453125	6.224898381282	157	27.685900753301
126	1.6484375	6.334384060404	156	28.374083896129
127	1.65234375	6.446597256401	163	29.082500565501
128	1.65625	6.561618325370	159	29.811830182679
129	1.66015625	6.679530326648	168	30.562777533029
130	1.6640625	6.800419123815	173	31.336073800869
131	1.66796875	6.924373489973	174	32.132477650687
132	1.671875	7.051485217256	174	32.952776355753
133	1.67578125	7.181849230881	181	33.797786976988
134	1.6796875	7.315563707853	177	34.668357594031
135	1.68359375	7.452730200703	187	35.565368591802
136	1.6875	7.593453766191	193	36.489734003891
137	1.69140625	7.737843099540	194	37.442402916982
138	1.6953125	7.886010674183	194	38.424360938139
139	1.69921875	8.038072887400	202	39.436631728499
140	1.703125	8.194150212046	199	40.480278606244
141	1.70703125	8.354367354771	210	41.556406222945
142	1.7109375	8.518853420800	216	42.666162315549
143	1.71484375	8.687742085843	218	43.810739539193
144	1.71875	8.861171775258	219	44.991377383685
145	1.72265625	9.039285850888	227	46.209364178233
146	1.7265625	9.2222332805873	225	47.466039188497
147	1.73046875	9.410166467940	236	48.762794811184
148	1.734375	9.603246211319	243	50.101078869751
149	1.73828125	9.801637177962	246	51.482397017709
150	1.7421875	10.005510508313	248	52.908315253800

Table 3 ($y' = x^3y$, seven nodes, 4 times) 4/4

S	x	y	E	y'
151	1.74609375	10.215043582126	256	54.380462555059
152	1.75	10.430420269795	255	55.900533633433
153	1.75390625	10.651831194787	267	57.470291822816
154	1.7578125	10.879474007477	275	59.091572101793
155	1.76171875	11.113553671202	278	60.766284260424
156	1.765625	11.354282760911	281	62.496416217308
157	1.76953125	11.601881775088	291	64.284037494917
158	1.7734375	11.856579461546	291	66.131302861119
159	1.77734375	12.118613157879	303	68.040456145971
160	1.78125	12.388229147030	313	70.013834241517
161	1.78515625	12.665683028998	317	72.053871295448
162	1.7890625	12.951240109251	321	74.163103107622
163	1.79296875	13.245175804733	332	76.3444171740180
164	1.796875	13.547776068297	333	78.599830352292
165	1.80078125	13.859337832561	347	80.932948271743
166	1.8046875	14.180169473929	358	83.346516314467
167	1.80859375	14.510591298012	363	85.843652366467
168	1.8125	14.850936047344	369	88.427607240886
169	1.81640625	15.201549432525	381	91.101770824921
170	1.8203125	15.562790687962	384	93.869678531883
171	1.82421875	15.935033153508	398	96.735018075047
172	1.828125	16.318664883088	411	99.761636579122
173	1.83203125	16.714089281916	418	102.773548048797
174	1.8359375	17.121725773564	426	105.954941212457
175	1.83984375	17.542010498420	439	109.250187761282
176	1.84375	17.975397045142	444	112.663851005070
177	1.84765625	18.422357216834	460	116.200694967677
178	1.8515625	18.883381833534	475	119.865693944567
179	1.85546875	19.358981573085	484	123.664042254901
180	1.859375	19.849687852212	494	127.601166272528
181	1.86328125	20.356053749875	509	131.682732587669
182	1.8671875	20.878654975105	517	135.914662622968
183	1.87109375	21.418090881683	535	140.303143441822
184	1.875	21.974985531888	553	144.854640957270
185	1.87890625	22.549988812111	563	149.575913519248
186	1.8828125	23.143777602874	576	154.474026210578
187	1.88671875	23.757057006125	594	159.556365891015
188	1.890625	24.390561632820	604	164.830657031248
189	1.89453125	25.045056954049	626	170.304978381270
190	1.8984375	25.721340718855	646	175.987780518612
191	1.90234375	26.420244442497	660	181.887904327331
192	1.90625	27.142634968783	676	188.014600459272
193	1.91015625	27.889416110389	698	194.377549833007
194	1.9140625	28.661530371391	711	200.986885229577
195	1.91796875	29.459960756461	736	207.853214047676
196	1.921875	30.285732671200	761	214.987642283091
197	1.92578125	31.139915918752	778	222.401799803936
198	1.9296875	32.023626797774	798	230.107866995063
199	1.93359375	32.938030307251	824	238.118602850253
200	1.9375	33.884342464025	842	246.447374596135
205	1.95703125	39.140982566079	979	293.376285332400
207	1.96484375	41.516014727168	1037	314.919618437490

Table 4 ($y' = 1/x$, seven nodes, 4 times) 1/2

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
1	0.8125	-0.2076393648	0	1.2307692308
2	0.875	-0.1335313926	0	1.1428571429
3	0.9375	-0.0645385211	0	1.0666666667
4	1	0.0000000000	0	1.0000000000
5	1.0625	0.0606246218	0	0.9411764706
6	1.125	0.1177830357	0	0.8888888889
7	1.1875	0.1718502569	0	0.8421052632
8	1.25	0.2231435537	24	0.8000000000
9	1.3125	0.2719337169	14	0.7619047619
10	1.375	0.3184537320	9	0.7272727273
11	1.4375	0.3629054942	5	0.6956521739
12	1.5	0.4054651085	3	0.6666666667
13	1.5625	0.4462871029	2	0.6400000000
14	1.625	0.4855078183	25	0.6153846154
15	1.6875	0.5232481453	15	0.5925925926
16	1.75	0.5596157889	9	0.5714285714
17	1.8125	0.5947071083	6	0.5517241379
18	1.875	0.6286086598	4	0.5333333333
19	1.9375	0.6613984825	2	0.5161290323
20	2.0	0.6931471831	25	0.5000000000
21	2.0625	0.7239188407	15	0.4848484848
22	2.125	0.7537718033	9	0.4705882353
23	2.1875	0.7827593398	6	0.4571428571
24	2.25	0.8109302166	4	0.4444444444
25	2.3125	0.8383291907	3	0.4324324324
26	2.375	0.8649974400	25	0.4210526316
27	2.4375	0.8909729254	15	0.4102564103
28	2.5625	0.9409833469	24	0.3902439024
29	2.6875	0.9886113948	13	0.3720930233
30	2.8125	1.0340737697	22	0.3555555556
31	2.9375	1.0775588805	10	0.3404255319
32	3.0625	1.1192315764	5	0.3265306122
33	3.1875	1.1592369122	17	0.3137254902
34	3.3125	1.1977031938	25	0.3018867925
35	3.4375	1.2347444644	14	0.2909090909
36	3.5625	1.2704625478	22	0.2807017544
37	3.6875	1.3049487227	11	0.2711864407
38	3.8125	1.3382851425	6	0.2622950820
39	3.9375	1.3705460059	17	0.2539682540
40	4.0625	1.4017985502	25	0.2461538462
41	4.1875	1.4321038986	14	0.2388059701
42	4.3125	1.4615177846	23	0.2318840580
43	4.4375	1.4900911559	11	0.2253521127
44	4.5625	1.5178707195	6	0.2191780822
45	4.6875	1.5448993930	17	0.2133333333
46	4.8125	1.5712167021	25	0.2077922078
47	4.9375	1.5968591317	14	0.2025316456
48	5.1875	1.6462518882	26	0.1927710843
49	5.4375	1.6933193991	27	0.1839080460
50	5.6875	1.7382707863	20	0.1758241758

Table 4 ($y' = 1/x$, seven nodes, 4 times) 2/2

$S = \text{step}$		$E = (\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
51	5.9375	1.7812881708	14	0.1684210526
52	6.1875	1.8225311299	20	0.1616161616
53	6.4375	1.8621402676	16	0.1553398058
54	6.6875	1.9002401150	28	0.1495327103
55	6.9375	1.9369414818	28	0.1441441441
56	7.1875	1.9723434082	21	0.1391304348
57	7.4375	2.0065347724	15	0.1344537815
58	7.6875	2.0395956351	20	0.1300813008
59	7.9375	2.0715983658	16	0.1259842520
60	8.1875	2.1026086037	28	0.1221374046
61	8.4375	2.1326860590	28	0.1185185185
62	8.6875	2.1618852130	21	0.1151079137
63	8.9375	2.1902559095	15	0.1118881119
64	9.1875	2.2178438665	20	0.1088435374
65	9.4375	2.2446911162	16	0.1059602649
66	9.6875	2.2708363975	28	0.1032258065
67	9.9375	2.2963154828	28	0.1006289308
68	10.4375	2.3454050932	30	0.0958083832
69	10.9375	2.3921972542	25	0.0914285714
70	11.4375	2.4368974339	33	0.0874316940
71	11.9375	2.4796847077	19	0.0837696335
72	12.4375	2.5207161043	19	0.0804020101
73	12.9375	2.5601300740	29	0.0772946860
74	13.4375	2.5980493090	31	0.0744186047
75	13.9375	2.6345830518	26	0.0717488789
76	14.4375	2.6698289917	34	0.0692640693
77	14.9375	2.7038748316	19	0.0669456067
78	15.4375	2.7367996163	19	0.0647773279
79	15.9375	2.7686748259	29	0.0627450980
80	16.4375	2.7995653130	31	0.0608365019
81	16.9375	2.8295301013	26	0.0590405904
82	17.4375	2.8586230630	34	0.0573476703
83	17.9375	2.8868934954	19	0.0557491289
84	18.4375	2.9143866360	19	0.0542372881
85	18.9375	2.9411440862	30	0.0528052805
86	19.4375	2.9672041930	31	0.0514469453
87	19.9375	2.9926023832	26	0.0501567398
88	20.4375	3.0173714521	34	0.0489296636
89	20.9375	3.0415418115	19	0.0477611940
90	21.4375	3.0651417268	19	0.0466472303
91	21.9375	3.0881975042	30	0.0455840456
92	22.4375	3.1107336693	31	0.0445682451
93	22.9375	3.1327731284	26	0.0435967302
94	23.4375	3.1543373071	34	0.0426666667
95	23.9375	3.1754462688	19	0.0417754569
96	24.4375	3.1961188396	19	0.0409207161
97	24.9375	3.2163726976	30	0.0401002506
98	25.4375	3.2362244663	31	0.0393120393
99	25.9375	3.2556898006	26	0.0385542169
100	26.4375	3.2747834602	34	0.0378250591
150	51.4375	3.9403674803	19	0.0194410693
200	76.4375	4.3364734165	31	0.0130825838
207	79.9375	4.3812450820	26	0.0125097733

Table 5 ($y' = x^3y$, nine nodes, 4 times) 1/4

$S = \text{step}$		$E = (\text{error of } y) \times 10^{12}$		
S	x	y	E	y'
1	-0.25	1.000977039492	0	-0.015640266242
2	-0.1875	1.000309038221	0	-0.006593833992
3	-0.125	1.000061037019	0	-0.001953244213
4	-0.0625	1.000003814705	0	-0.000244141556
5	0	1.000000000000	0	0.000000000000
6	0.0625	1.000003814705	0	0.000244141556
7	0.125	1.000061037019	0	0.001953244213
8	0.1875	1.000309038221	0	0.006593833992
9	0.25	1.000977039492	0	0.015640266242
10	0.3125	1.002387030245	23	0.030590424507
11	0.375	1.004956088681	46	0.052995731239
12	0.4375	1.009201161005	71	0.084510741754
13	0.5	1.015747708689	102	0.126968463586
14	0.53125	1.020112543721	47	0.152948392557
15	0.5625	1.025344064331	24	0.182489214574
16	0.59375	1.031558688000	48	0.215925935089
17	0.625	1.038883909234	47	0.253633766903
18	0.65625	1.047459683459	48	0.296036502945
19	0.6875	1.057440095165	74	0.343616398112
20	0.71875	1.068995362258	50	0.396925859759
21	0.75	1.082314239135	106	0.456601319635
22	0.78125	1.097606892423	53	0.523379751407
23	0.8125	1.115108342808	28	0.598118415319
24	0.84375	1.135082582440	56	0.681818556829
25	0.875	1.157827506807	54	0.775653974286
26	0.890625	1.170342770906	31	0.826794772234
27	0.90625	1.183680830269	55	0.881005608198
28	0.921875	1.197891286240	32	0.938498361498
29	0.9375	1.213027194716	30	0.999503608927
30	0.953125	1.229145372318	33	1.064272482888
31	0.96875	1.246306733512	59	1.133078732241
32	0.984375	1.264576661423	34	1.206221010051
33	1.0	1.284025416745	57	1.284025416745
34	1.015625	1.304728587953	36	1.366848329416
35	1.03125	1.326767588928	59	1.455079554544
36	1.046875	1.350230207905	39	1.549145843583
37	1.0625	1.375211215160	34	1.649514819356
38	1.078125	1.401813035071	40	1.756699364998
39	1.09375	1.430146491036	64	1.871262536718
40	1.109375	1.460331631539	42	1.993823068907
41	1.125	1.492498647667	63	2.125061551073
42	1.140625	1.526788892423	45	2.265727365737
43	1.15625	1.563356015712	65	2.416646492428
44	1.171875	1.602367228148	50	2.578730294704
45	1.1875	1.644004711291	41	2.752985428404
46	1.203125	1.688467192058	52	2.940525026672
47	1.21875	1.735971703186	73	3.142581343423
48	1.234375	1.786755553954	54	3.360520063652
49	1.25	1.841078539268	75	3.595856522008
50	1.265625	1.899225418390	59	3.850274107263

Table 5 ($y' = x^3y$, nine nodes, 4 times) 2/4

$S = \text{step}$		$E = (\text{error of } y) \times 10^{12}$		
S	x	y	E	y'
51	1.28125	1.961508700983	77	4.125645177626
52	1.296875	2.028271781393	67	4.424054859418
53	1.3125	2.099892470599	54	4.747828166558
54	1.328125	2.176786980794	71	5.099560945817
55	1.3359375	2.217353581484	72	5.286805996431
56	1.34375	2.259414427171	87	5.482155238681
57	1.3515625	2.303034446177	75	5.686025907304
58	1.359375	2.348281921291	77	5.898859748900
59	1.3671875	2.395228687321	79	6.121124623351
60	1.375	2.443950341782	65	6.353316220530
61	1.3828125	2.494526469487	82	6.595959900844
62	1.390625	2.547040881816	83	6.849612668659
63	1.3984375	2.601581872261	84	7.114865290485
64	1.40625	2.658242488905	101	7.392344567916
65	1.4140625	2.717120825311	89	7.682715778256
66	1.421875	2.778320331378	91	7.986685296771
67	1.4296875	2.841950145144	94	8.305003413755
68	1.4375	2.908125447620	80	8.638467363572
69	1.4453125	2.976967842178	98	8.987924582354
70	1.453125	3.048605760110	99	9.354276212253
71	1.4609375	3.123174894925	101	9.738480874262
72	1.46875	3.200818667009	119	10.141558730007
73	1.4765625	3.281688721224	107	10.564595857261
74	1.484375	3.365945460231	111	11.008748966086
75	1.4921875	3.453758615755	115	11.475250482426
76	1.5	3.545307861324	100	11.965414031967
77	1.5078125	3.640783469539	120	12.480640357577
78	1.515625	3.740387017131	122	13.022423706383
79	1.5234375	3.844332142242	124	13.592358728985
80	1.53125	3.952845357496	145	14.192147932862
81	1.5390625	4.066166923617	132	14.823609738978
82	1.546875	4.184551788779	137	15.488687195209
83	1.5546875	4.308270598404	143	16.189457401997
84	1.5625	4.437610781849	128	16.928141715428
85	1.5703125	4.572877722070	151	17.707116795884
86	1.578125	4.714396014865	154	18.528926576659
87	1.5859375	4.862510825888	156	19.396295237865
88	1.59375	5.017589352958	181	20.312141273780
89	1.6015625	5.180022402854	167	21.279592754072
90	1.609375	5.350226092650	175	22.302003889248
91	1.6171875	5.528643685532	184	23.382973017329
92	1.625	5.715747573399	169	24.526362146013
93	1.6328125	5.912041418699	196	25.736318193879
94	1.640625	6.118062469026	200	27.017296088052
95	1.6484375	6.334384060452	203	28.374083896343
96	1.65625	6.561618325443	232	29.811830183015
97	1.6640625	6.800419123862	220	31.336073801085
98	1.671875	7.051485217313	231	32.952776356022
99	1.6796875	7.315563707920	244	34.668357594346
100	1.6875	7.593453766227	229	36.489734004066

Table 5 ($y' = x^3y$, nine nodes, 4 times) 3/4

S	x	y	E	y'
101	1.6953125	7.886010674249	261	38.424360938464
102	1.703125	8.194150212114	268	40.480278606584
103	1.7109375	8.518853420857	273	42.666162315837
104	1.71875	8.861171775349	309	44.991377384144
105	1.7265625	9.222232805946	298	47.466039188871
106	1.734375	9.603246211390	314	50.101078870122
107	1.7421875	10.005510508398	333	52.908315254253
108	1.75	10.430420269860	320	55.900533633778
109	1.7578125	10.879474007562	360	59.091572102256
110	1.765625	11.354282761001	372	62.496416217806
111	1.7734375	11.856579461636	381	66.131302861619
112	1.78125	12.388229147145	427	70.013834242161
113	1.78515625	12.665683029097	415	72.053871296007
114	1.7890625	12.951240109358	428	74.163103108235
115	1.79296875	13.245175804836	435	76.344171740772
116	1.796875	13.547776068407	443	78.599830352928
117	1.80078125	13.859337832670	455	80.932948272376
118	1.8046875	14.180169474028	457	83.346516315049
119	1.80859375	14.510591298125	476	85.843652367134
120	1.8125	14.850936047483	508	88.427607241715
121	1.81640625	15.201549432643	498	91.101770825626
122	1.8203125	15.562790688093	514	93.869678532670
123	1.82421875	15.935033153633	524	96.735018075810
124	1.828125	16.318664883210	533	99.701636579868
125	1.83203125	16.714089282049	550	102.773548049611
126	1.8359375	17.121725773691	553	105.954941213246
127	1.83984375	17.542010498556	576	109.250187762131
128	1.84375	17.975397045309	611	112.663851006117
129	1.84765625	18.442357216978	604	116.200694968585
130	1.8515625	18.883381833682	623	119.865693945507
131	1.85546875	19.358981573239	637	123.664042549993
132	1.859375	19.849687852367	649	127.601166273525
133	1.86328125	20.356053750036	671	131.682732588715
134	1.8671875	20.878654975265	676	135.914662624007
135	1.87109375	21.418090881851	703	140.303143442923
136	1.875	21.974985532078	743	144.854640958525
137	1.87890625	22.549988812287	740	149.575913520419
138	1.8828125	23.143777603062	764	154.474026211830
139	1.88671875	23.757057006313	783	159.556365892280
140	1.890625	24.390561633013	797	164.830657032552
141	1.89453125	25.045056954250	826	170.304978382635
142	1.8984375	25.721340719044	835	175.987780519903
143	1.90234375	26.420244442705	868	181.887904328763
144	1.90625	27.142634969021	914	188.014600460920
145	1.91015625	27.889416110607	915	194.377549834523
146	1.9140625	28.661530371626	946	200.986885231223
147	1.91796875	29.459960756697	971	207.853214049336
148	1.921875	30.285732671430	990	214.987642284722
149	1.92578125	31.139915919002	1028	222.401799805725
150	1.9296875	32.023626798018	1042	230.107866996817

Table 5 ($y' = x^3y$, nine nodes, 4 times) 4/4

S	x	y	E	y'
151	1.93359375	32.938030307510	1082	238.118602852123
152	1.9375	33.884342464320	1136	246.447374598280
153	1.94140625	34.863832739508	1144	255.108188938684
154	1.9453125	35.877826620291	1184	264.115725013200
155	1.94921875	36.927708303799	1218	273.485369168628
156	1.953125	38.014923530777	1244	283.233251651948
157	1.95703125	39.140982566393	1293	293.376285334753
158	1.9609375	40.307463336371	1314	303.932206588430
159	1.96484375	41.516014727496	1365	314.919618439974
160	1.96875	42.768360060997	1430	326.358036138062
161	1.97265625	44.066300749074	1446	338.267935276563
162	1.9765625	45.411720144816	1498	350.670802627386
163	1.98046875	46.806587595761	1544	363.589189839109
164	1.984375	48.252962713543	1580	377.046770179956
165	1.98828125	49.752999871256	1644	391.068398503019
166	1.9921875	51.308952941537	1676	405.680174627768
167	1.99609375	52.923180289587	1740	420.909510348022
168	2.0	54.598150034965	1821	436.785200279719
169	2.00390625	56.336445598227	1849	453.337496786789
170	2.0078125	58.140771548814	1917	470.598189233352
171	2.01171875	60.013959770862	1979	488.600687820720
172	2.015625	61.958975966408	2030	507.380112298399
173	2.01953125	63.978926514947	2113	526.973385843283
174	2.0234375	66.077065710140	2162	547.419334425730
175	2.02734375	68.256803396361	2244	568.758792006782
176	2.03125	70.521713027656	2346	591.034711920781
177	2.03515625	72.875540174756	2393	614.292284834390
178	2.0390625	75.322211506578	2484	638.579063692249
179	2.04296875	77.865844273578	2568	663.945096079993
180	2.046875	80.510756323935	2639	690.443064479951
181	2.05078125	83.261476683624	2750	718.128434910476
182	2.0546875	86.122756734137	2822	747.059614478559
183	2.05859375	89.099582024466	2930	777.298118416693
184	2.0625	92.197184754514	3062	808.908747197992
185	2.06640625	95.421056971368	3134	841.959774379499
186	2.0703125	98.776964521640	3257	876.523145859729
187	2.07421875	102.270961804991	3373	912.674691275060
188	2.078125	105.909407379009	3473	950.494348327369
189	2.08203125	109.698980466743	3623	990.066400870575
190	2.0859375	113.646698422343	3727	1031.479731646882
191	2.08984375	117.759935214672	3873	1074.828090631866
192	2.09375	122.046440990635	4047	1120.210379994700
193	2.09765625	126.514362786165	4156	1167.730956768171
194	2.1015625	131.172266456246	4325	1217.499954389514
195	2.10546875	136.029159899102	4485	1269.633624344008
196	2.109375	141.094517657020	4630	1324.254699250755
197	2.11328125	146.378306979411	4833	1381.492778801152
198	2.1171875	151.891015440235	4986	1441.484740066743
199	2.12109375	157.643680209085	5185	1504.375173808385
200	2.125	163.647919079462	5419	1570.316848510543
205	2.3671875	2565.780107192876	413002	34034.346044523740
209	2.3828125	3163.202301679539	509246	42795.349457116334

Table 6 ($y' = 1/x$, nine nodes, 4 times) 1/2

S	x	y	E	y'
1	0.75	-0.287682072452	0	1.333333333333
2	0.8125	-0.207639364778	0	1.230769230769
3	0.875	-0.133531392625	0	1.142857142857
4	0.9375	-0.064538521138	0	1.666666666667
5	1	0.000000000000	0	1.000000000000
6	1.0625	0.060624621816	0	0.941176470588
7	1.125	0.117783035656	0	0.888888888889
8	1.1875	0.171850256927	0	0.842105263158
9	1.25	0.223143551314	0	0.800000000000
10	1.3125	0.271933716109	626	0.761904761905
11	1.375	0.318453731447	328	0.727272727273
12	1.4375	0.362905493868	178	0.695652173913
13	1.5	0.405465108208	100	0.666666666667
14	1.5625	0.446287102686	58	0.640000000000
15	1.625	0.485507815816	34	0.615384615385
16	1.6875	0.523248143786	21	0.592592592593
17	1.75	0.559615787948	13	0.571428571429
18	1.8125	0.594707108381	634	0.551724137931
19	1.875	0.628608659756	333	0.533333333333
20	1.9375	0.661398482427	182	0.516129032258
21	2.0	0.693147180663	103	0.500000000000
22	2.0625	0.723918839286	60	0.484848484848
23	2.125	0.753771802412	36	0.470588235294
24	2.1875	0.782759339271	22	0.457142857143
25	2.25	0.810930216230	14	0.444444444444
26	2.3125	0.838329191039	634	0.432432432432
27	2.375	0.864997437820	334	0.421052631579
28	2.4375	0.890972924072	182	0.410256410256
29	2.5	0.916290731977	103	0.400000000000
30	2.625	0.965080896704	660	0.380952380952
31	2.75	1.011600912020	341	0.363636363636
32	2.875	1.056052674761	512	0.347826086957
33	3.0	1.098612288871	203	0.333333333333
34	3.125	1.139434283282	94	0.320000000000
35	3.25	1.178654996390	48	0.307692307692
36	3.375	1.216395324679	355	0.296296296296
37	3.5	1.252762968611	116	0.285714285714
38	3.625	1.287854288975	668	0.275862068966
39	3.75	1.321755840329	346	0.266666666667
40	3.875	1.354545663321	515	0.258064516129
41	4.0	1.386294361325	205	0.250000000000
42	4.125	1.417066019882	95	0.242424242424
43	4.25	1.446918982985	49	0.235294117647
44	4.375	1.475906520165	355	0.228571428571
45	4.5	1.504077396893	116	0.222222222222
46	4.625	1.531476371633	669	0.216216216216
47	4.75	1.558144618393	347	0.210526315789
48	4.875	1.584120104965	515	0.205128205128
49	5.0	1.609437912639	205	0.200000000000
50	5.25	1.658228077277	674	0.190476190476

Table 6 ($y' = 1/x$, nine nodes, 4 times) 2/2

S	x	y	E	y'
51	5.5	1.704748092682	444	0.181818181818
52	5.75	1.749199855334	525	0.173913043478
53	6.0	1.791759469533	305	0.166666666667
54	6.25	1.832581463855	107	0.160000000000
55	6.5	1.871802177052	151	0.153846153846
56	6.75	1.909542505252	368	0.148148148148
57	7.0	1.945910149274	218	0.142857142857
58	7.25	1.981001469549	682	0.137931034483
59	7.5	2.014903020991	449	0.133333333333
60	7.75	2.047692843894	528	0.129032258065
61	8.0	2.079441541988	308	0.125000000000
62	8.25	2.110213200455	109	0.121212121212
63	8.5	2.140066163648	152	0.117647058824
64	8.75	2.169053700738	368	0.114285714286
65	9.0	2.197224577555	219	0.111111111111
66	9.25	2.224623552207	682	0.108108108108
67	9.5	2.251291799056	449	0.105263157895
68	9.75	2.277267285538	529	0.102564102564
69	10.0	2.302585093302	308	0.100000000000
70	10.5	2.351375257940	776	0.095238095238
71	11.0	2.397895273345	546	0.090909090909
72	11.5	2.442347035997	628	0.086956521739
73	12.0	2.484906650196	408	0.083333333333
74	12.5	2.525728644518	210	0.080000000000
75	13.0	2.564949357715	253	0.076923076923
76	13.5	2.602689685915	470	0.074074074074
77	14.0	2.639057329936	321	0.071428571429
78	14.5	2.674148650211	785	0.068965517241
79	15.0	2.708050201654	552	0.066666666667
80	15.5	2.740840024556	631	0.064516129032
81	16.0	2.772588722650	410	0.062500000000
82	16.5	2.803360381118	211	0.060606060606
83	17.0	2.833213344311	254	0.058823529412
84	17.5	2.862200881401	471	0.057142857143
85	18.0	2.890371758218	321	0.055555555556
86	18.5	2.917770732869	785	0.054054054054
87	19.0	2.944438979718	552	0.052631578947
88	19.5	2.970414466201	631	0.051282051282
89	20.0	2.995732273964	410	0.050000000000
90	20.5	3.020424886356	212	0.048780487805
91	21.0	3.044522437978	255	0.047619047619
92	21.5	3.068052935605	471	0.046511627907
93	22.0	3.091042453680	322	0.045454545455
94	22.5	3.113515309996	785	0.044444444444
95	23.0	3.135494216481	552	0.043478260870
96	23.5	3.157000421781	631	0.042553191489
97	24.0	3.178053830758	410	0.041666666667
98	24.5	3.198673117762	212	0.040816326531
99	25.0	3.218875825123	255	0.040000000000
100	25.5	3.238678452636	471	0.039215686275
105	28.0	3.332204510586	410	0.035714285714
109	30.0	3.401197381984	322	0.033333333333

Table 7 ($y' = x^3y$, five nodes, 5 times) 1/4

$S=\text{step}$		$E=(\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
1	-0.125	1.0000610370	0	-0.0019532442
2	-0.0625	1.0000038147	0	-0.0002441416
3	0	1.0000000000	0	0.0000000000
4	0.0625	1.0000038147	0	0.0002441416
5	0.125	1.0000610370	0	0.0019532442
6	0.1875	1.0003090407	25	0.0065938340
7	0.25	1.0009770445	50	0.0156402663
8	0.3125	1.0023870378	76	0.0305904247
9	0.375	1.0049560970	103	0.0845107431
10	0.4375	1.0092011770	161	0.0845107431
11	0.5	1.0157477314	228	0.1269684664
12	0.53125	1.0201125592	156	0.1529483949
13	0.5625	1.0253440809	166	0.1824892175
14	0.59375	1.0315587038	158	0.2159259384
15	0.625	1.0388839327	235	0.2536337726
16	0.65625	1.0474596999	165	0.2960365076
17	0.6875	1.0574401127	176	0.3436164038
18	0.71875	1.0689953793	171	0.3969258661
19	0.75	1.0823142640	249	0.4566013301
20	0.78125	1.0976069107	183	0.5233797601
21	0.796875	1.1060657762	253	0.5596951724
22	0.8125	1.1151083685	257	0.5981184291
23	0.828125	1.1247686299	258	0.6387793706
24	0.84375	1.1350826015	191	0.6818185683
25	0.859375	1.1460886382	262	0.7273883712
26	0.875	1.1578275334	266	0.7756539921
27	0.890625	1.1703427976	267	0.8267947911
28	0.90625	1.1836808504	202	0.8810056232
29	0.921875	1.1978913135	273	0.9384983828
30	0.9375	1.2130272226	279	0.9995036319
31	0.953125	1.2291454003	280	1.0642725071
32	0.96875	1.2463067551	216	1.1330787518
33	0.984375	1.2645766901	287	1.0262210374
34	1.0	1.2840254462	295	1.2840254462
35	1.015625	1.3047286176	296	1.3668483604
36	1.03125	1.3267676123	234	1.4550795802
37	1.046875	1.3502302384	305	1.5491458786
38	1.0625	1.3752112467	316	1.6495148572
39	1.078125	1.4018130668	318	1.7566994047
40	1.09375	1.4301465167	258	1.8712625704
41	1.109375	1.4603316644	329	1.9938231138
42	1.125	1.4924986818	342	2.1250615997
43	1.1328125	1.5093690997	340	2.1941663606
44	1.140625	1.5267889268	344	2.2657274168
45	1.1484375	1.5447777620	348	2.3398504690
46	1.15625	1.5633560515	358	2.4166465477
47	1.1640625	1.5825451286	357	2.4962323072
48	1.171875	1.6023672643	362	2.5787303528
49	1.1796875	1.6228457095	366	2.6642695705
50	1.1875	1.6440047489	377	2.7529854914

Table 7 ($y' = x^3y$, five nodes, 5 times) 2/4

$S=\text{step}$		$E=(\text{error of } y) \times 10^{10}$		
S	x	y	E	y'
51	1.1953125	1.6658697510	376	2.8450206686
52	1.2031255	1.6884672301	381	2.9405250929
53	1.2109375	1.7118249004	386	3.0396566158
54	1.21875	1.7359717429	398	3.1425814153
55	1.2265625	1.7609380683	398	3.2494744796
56	1.2343755	1.7867555942	403	3.3605201394
57	1.2421875	1.8134575144	409	3.4759126129
58	1.25	1.8410785813	421	3.5958566042
59	1.2578125	1.8696551877	422	3.7205679281
60	1.2656255	1.8992254612	429	3.8502741941
61	1.2734275	1.9298293538	436	3.9852155135
62	1.28125	1.9615087458	449	4.1256452718
63	1.2890625	1.9943075492	451	4.2718309400
64	1.2968755	2.0282718271	458	4.4240549592
65	1.3046875	2.0634499083	466	4.5826156617
66	1.3125	2.0998925185	480	4.7478282750
67	1.3203125	2.1376529138	483	4.9200259795
68	1.328125	2.1767870299	492	5.0995610608
69	1.3359375	2.2173536315	501	5.2868061157
70	1.34375	2.2594144787	516	5.4821553637
71	1.3515625	2.3030344982	521	5.6860260357
72	1.359375	2.3482819742	531	5.8988598820
73	1.3671875	2.3952287414	542	6.1211247615
74	1.375	2.4439503975	558	6.3533163654
75	1.3828125	2.4945265258	564	6.5959600498
76	1.390625	2.5470409393	576	6.8496128232
77	1.3984375	2.6015819310	588	7.1148654512
78	1.40625	2.6582425495	607	7.3923447363
79	1.4140625	2.7171208867	615	7.6827159519
80	1.421875	2.7783203941	628	7.9866854771
81	1.4296875	2.8419502094	643	8.3050036014
82	1.4375	2.9081255139	663	8.6384675604
83	1.4453125	2.9769679095	674	8.9879247855
84	1.453125	3.0486058289	690	9.3542764235
85	1.4609375	3.1231749655	707	9.7384810944
86	1.46875	3.2008187399	730	10.1415589609
87	1.4765625	3.2816887955	743	10.5645960962
88	1.484375	3.3659455363	762	11.0087492148
89	1.4921875	3.4537586939	782	11.4752507420
90	1.5	3.5453079421	808	11.9654143044
91	1.5078125	3.6407835519	825	12.4806406400
92	1.515625	3.7403871017	847	13.0224240008
93	1.5234375	3.8443322293	871	13.5923590367
94	1.52734375	3.8980031433	883	13.6576023444
95	1.53125	3.9528454469	895	14.1921482537
96	1.53515625	4.0088896088	908	14.5038016758
97	1.5390625	4.0661670157	922	14.8236100745
98	1.54296875	4.1247100020	934	15.1518202921
99	1.546875	4.1845518834	948	15.4886875454
100	1.55078125	4.2457269888	961	15.8344757342

Table 7 ($y' = x^3y$, five nodes, 5 times) 3/4

S =step	$E=(\text{error of } y) \times 10^{10}$			
S	x	y	E	y'
101	1.5546875	4.3082706959	977	16.1894577684
102	1.55859375	4.3722194659	990	16.5539159039
103	1.5625	4.4376108822	1005	16.9281420983
104	1.56640625	4.5044836877	1020	17.3124383728
105	1.5703125	4.5728778256	1036	17.7071171966
106	1.57421875	4.6428344802	1051	18.1125018808
107	1.578125	4.7143961215	1068	18.5289269956
108	1.58203125	4.7876065483	1084	18.9567387970
109	1.5859375	4.8625109359	1102	19.3962956769
110	1.58984375	4.9391558838	1118	19.8479686278
111	1.59375	5.0175894664	1136	20.3121417330
112	1.59765625	5.0978612845	1154	20.7892126690
113	1.6015625	5.1800225201	1174	21.2795932357
114	1.60546875	5.2641259916	1192	21.7837099041
115	1.609375	5.3502262136	1212	22.3020043935
116	1.61328125	5.4383794567	1232	22.8349342650
117	1.6171875	5.5286438106	1253	23.3829735464
118	1.62109375	5.6210792500	1273	23.9466133795
119	1.625	5.7157477027	1294	24.5263627007
120	1.62890625	5.8127131204	1316	25.1227489433
121	1.6328125	5.9120415525	1340	25.7363187763
122	1.63671875	6.1308012222	1362	26.3676388694
123	1.640625	6.1180626074	1385	27.0172966990
124	1.64453125	6.2248985221	1410	27.6859013796
125	1.6484375	6.3343842038	1435	28.3740845384
126	1.65234375	6.4465974022	1460	29.0825012234
127	1.65625	6.5616184738	1486	29.8118308571
128	1.66015625	6.6796304777	1513	30.5627782244
129	1.6640625	6.8004192777	1541	31.3360745102
130	1.66796875	6.9243736466	1568	32.1324783776
131	1.671875	7.0514853768	1597	32.9527771012
132	1.67578125	7.1818493933	1626	33.7977877416
133	1.6796875	7.3155638735	1658	34.6683583788
134	1.68359375	7.4527303693	1688	35.5653693964
135	1.6875	7.5934539380	1720	36.4897348293
136	1.69140625	7.7378432746	1752	37.4424037640
137	1.6953125	7.8860108527	1787	38.4243618079
138	1.69921875	8.0380730692	1820	39.4366326207
139	1.703125	8.1941503974	1856	40.4802795220
140	1.70703125	8.3543675438	1892	41.5564071631
141	1.7109375	8.5188536136	1930	42.6661632813
142	1.71484375	8.6877422824	1968	43.8107405304
143	1.71875	8.8611719757	2007	44.9913784015
144	1.72265625	9.0392860554	2047	46.2093652237
145	1.7265625	9.2222330146	2090	47.4660402629
146	1.73046875	9.4101666808	2131	48.7627959144
147	1.734375	9.6032464286	2175	50.1010800032
148	1.73828125	9.8016373997	2220	51.4823981825
149	1.7421875	10.0055107348	2267	52.9083164513
150	1.74609375	10.2150438132	2314	54.3804637854

Table 7 ($y' = x^3y$, five nodes, 5 times) 4/4

S =step	$E=(\text{error of } y) \times 10^{10}$			
S	x	y	E	y'
151	1.75	10.4304205058	2362	55.9005348982
152	1.75390625	10.6518314358	2413	57.4702931231
153	1.7578125	10.8794742537	2465	59.0915734392
154	1.76171875	11.1135539227	2517	60.7662856353
155	1.765625	11.3542830178	2572	62.4964176313
156	1.76953125	11.6018820376	2628	64.2840389494
157	1.7734375	11.8565797299	2687	66.1313043579
158	1.77734375	12.1186134321	2745	68.0404576855
159	1.78125	12.3882294273	2806	70.0138358256
160	1.78515625	12.6656833156	2869	72.0538729258
161	1.7890625	12.9512404024	2935	74.1631047862
162	1.79296875	13.2451761044	3000	76.3441734676
163	1.796875	13.5477763748	3069	78.5993213074
164	1.80078125	13.8593381462	3140	80.9329501031
165	1.8046875	14.1801697949	3213	83.3465182009
166	1.80859375	14.5105916264	3287	85.8436543090
167	1.8125	14.8509363834	3364	88.4276092418
168	1.81640625	15.2015497765	3444	91.1017728864
169	1.8203125	15.5627910402	3526	93.8696806566
170	1.82421875	15.9350335141	3610	96.7350202641
171	1.828125	16.3186652523	3697	99.7016388351
172	1.83203125	16.7140896601	3786	102.7735503745
173	1.8359375	17.1217261611	3880	105.9549436107
174	1.83984375	17.5420108954	3974	109.2501902336
175	1.84375	17.9753974519	4072	112.6638535546
176	1.84765625	18.4223576337	4174	116.2006975973
177	1.8515625	18.8833822610	4279	119.8656966577
178	1.85546875	19.3589820112	4386	123.6640453477
179	1.859375	19.8496883014	4497	127.6011691602
180	1.86328125	20.3560542106	4612	131.6827355678
181	1.8671875	20.8786554477	4731	135.9146656995
182	1.87109375	21.4180913664	4853	140.3031466172
183	1.875	21.9749860292	4979	144.8546442355
184	1.87890625	22.5499893225	5109	149.5759169046
185	1.8828125	23.1437781268	5245	154.4740297073
186	1.88671875	23.7570575438	5383	159.5563695023
187	1.890625	24.3905621848	5526	164.8306607618
188	1.89453125	25.0450575209	5675	170.3049822360
189	1.896484375	25.3804246951	5751	173.1197800853
190	1.8984375	25.7213413010	5828	175.9877845017
191	1.900390625	25.0679123449	5906	178.9101129364
192	1.90234375	26.4202450405	5987	181.8879084442
193	1.904296875	26.7784488593	6067	184.9223403127
194	1.90625	27.1426355831	6150	188.0146047146
195	1.908203125	27.5129193564	6234	191.1659253700
196	1.91015625	27.8894167416	6319	194.3775542325
197	1.912109375	28.2722467741	6406	197.6507721881
198	1.9140625	28.6615310201	6494	200.9868897785
199	1.916015625	29.0573936349	6584	204.3872479376
200	1.91796875	29.4599614233	6675	207.8532187522
201	1.919921875	29.8693639006	6768	211.3862062393
203	1.923828125	30.7092049198	6958	218.6590117822
205	1.927734375	31.5780094746	7155	226.2175662741

Table 8 ($y' = x^3y$, seven nodes, 5 times) 1/4 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

S	x	y	E	y'
1	-0.1875	1.000309038221	0	-0.006593833992
2	-0.125	1.000061037019	0	-0.001953244213
3	-0.0625	1.000003814705	0	-0.000244141556
4	0	1.000000000000	0	0.000000000000
5	0.0625	1.000003814705	0	0.000244141556
6	0.125	1.000061037019	0	0.001953244213
7	0.1875	1.000309038221	0	0.006593833992
8	0.25	1.000977039504	12	0.015640266242
9	0.3125	1.002387030274	52	0.030590424508
10	0.375	1.004956088785	150	0.052995731245
11	0.4375	1.009201161273	339	0.084510741777
12	0.5	1.015747709245	659	0.126968463656
13	0.5625	1.025344065474	1166	0.182489214778
14	0.625	1.038883911151	1965	0.253633767371
15	0.65625	1.047459684126	716	0.296036503134
16	0.6875	1.057440095792	701	0.343616398315
17	0.71875	1.068995362947	739	0.396925860015
18	0.75	1.082314240257	1228	0.456601320108
19	0.78125	1.097606893184	814	0.523379751770
20	0.8125	1.115108344838	2058	0.598118416408
21	0.84375	1.135082583244	859	0.681818557312
22	0.875	1.157827507580	827	0.775653974804
23	0.90625	1.183680831147	933	0.881005608851
24	0.9375	1.213027196101	1416	0.999503610069
25	0.96875	1.246306734531	1077	1.133078733167
26	0.984375	1.264576662399	1011	1.206221010982
27	1.0	1.284025417702	1014	1.284025417702
28	1.015625	1.304728588957	1040	1.366848330468
29	1.03125	1.326767590380	1511	1.455079561366
30	1.046875	1.350230208957	1091	1.549145844789
31	1.0625	1.375211216310	1184	1.649514820735
32	1.078125	1.401813036169	1138	1.756699366374
33	1.09375	1.430146492106	1134	1.871262538118
34	1.109375	1.460331632678	1181	1.993823070462
35	1.125	1.492498649256	1652	2.125061553335
36	1.140625	1.526788893629	1252	2.265727367527
37	1.15625	1.563356016988	1341	2.416646494400
38	1.171875	1.602367229422	1324	2.578730296754
39	1.1875	1.644004712563	1313	2.752985430534
40	1.203125	1.688467193393	1386	2.940525028996
41	1.21875	1.735971704974	1861	3.142581346660
42	1.234375	1.786755555388	1488	3.360520066349
43	1.25	1.841078540771	1577	3.595856524943
44	1.265625	1.899225419929	1598	3.850274110383
45	1.28125	1.961508702498	1584	4.125645180796
46	1.296875	2.028271783019	1693	4.424054862966
47	1.3125	2.099892472725	2180	4.747828171365
48	1.328125	2.176786982568	1844	5.099560949971
49	1.34375	2.259414429026	1942	5.482155243181
50	1.359375	2.348281923229	2015	5.898859753769
51	1.375	2.443950343725	2008	6.353316225581
52	1.390625	2.547040883903	2169	6.849612674270

Table 8 ($y' = x^3y$, seven nodes, 5 times) 2/4 $S=\text{step}$ $E=(\text{error of } y) \times 10^{12}$

S	x	y	E	y'
53	1.3987475	2.061581874406	2229	7.114865296350
54	1.40625	2.658242491085	2281	7.392344573979
55	1.4140625	2.717120827551	2329	7.682715784590
56	1.421875	2.778320333580	2294	7.986685303103
57	1.4296875	2.841950147483	2436	8.305003205912
58	1.4375	2.908125450018	2478	8.638467370696
59	1.4453125	2.976967844627	2547	8.987924589746
60	1.453125	3.048605762626	2615	9.354276219972
61	1.4609375	3.123174897497	2673	9.738480882282
62	1.46875	3.200818669544	2654	10.141558738039
63	1.4765625	3.281688723924	2807	10.564595865953
64	1.484375	3.365945462990	2870	11.008748975110
65	1.4921875	3.453758618591	2950	11.475250491847
66	1.5	3.545307864263	3039	11.965414041888
67	1.5078125	3.640783472532	3113	12.480640367836
68	1.515625	3.740387020124	3115	13.022423716802
69	1.5234375	3.844332145403	3284	13.592358740161
70	1.53125	3.952845360722	3371	14.192147944445
71	1.5390625	4.066166926955	3470	14.823609751147
72	1.546875	4.184551792227	3585	15.488687207971
73	1.5546875	4.308270601941	3680	16.189457415287
74	1.5625	4.437610785430	3709	16.928141729087
75	1.5703125	4.572877725823	3904	17.707116810414
76	1.578125	4.714396018732	4021	18.528926591857
77	1.5859375	4.862510829878	4146	19.396295253779
78	1.59375	5.017589357074	4297	20.312141290443
79	1.6015625	5.180022407108	4421	21.279592771547
80	1.609375	5.350226096961	4487	22.302003907219
81	1.6171875	5.528643690064	4717	23.382973036499
82	1.625	5.715747578106	4875	24.526362166208
83	1.6328125	5.912041423541	5037	25.736318214956
84	1.640625	6.118062474062	5236	27.017296110291
85	1.6484375	6.334384065653	5404	28.374083919639
86	1.65625	6.561618330730	5519	29.811830207036
87	1.6640625	6.800419129442	5800	31.336073826798
88	1.671875	7.051485223097	6015	32.952776383053
89	1.6796875	7.315563713907	6231	34.668357622721
90	1.6875	7.593453772494	6495	36.489734034178
91	1.6953125	7.886010680714	6726	38.424360969963
92	1.703125	8.194150218758	6911	40.480278639403
93	1.7109375	8.518853427848	7264	42.666162350849
94	1.71875	8.861171782599	7560	44.991377420958
95	1.7265625	9.222232813503	7855	47.466029227769
96	1.734375	9.603246219287	8211	50.101078911322
97	1.7421875	10.005510516599	8533	52.908315297616
98	1.75	10.430420278359	8819	55.900533679332
99	1.7578125	10.879474016480	9278	59.091572150692
100	1.765625	11.354282770319	9690	62.496416269093
101	1.7734375	11.856579471357	10102	66.131302915841
102	1.78125	12.388229157309	10591	70.013834299606
103	1.7890625	12.951240119979	11049	74.163103169052
104	1.796875	13.547776079446	11483	78.599830416976
105	1.8046875	14.180169485670	12098	83.346516383472

Table 8 ($y' = x^3y$, seven nodes, 5 times) 3/4

S =step	$E=(\text{error of } y) \times 10^{12}$			
106	1.80859375	14.510591310025	12376	85.843652437537
107	1.8125	14.850936059644	12669	88.427607314126
108	1.81640625	15.201549445111	12966	91.101770900345
109	1.8203125	15.562790700779	13200	93.869678609189
110	1.82421875	15.935033166699	13590	96.735018155127
111	1.828125	16.318664896598	13921	99.701636661665
112	1.83203125	16.714089295751	14252	102.773548133865
113	1.8359375	17.121725787744	14605	105.954941300208
114	1.83984375	17.542010512941	14960	109.250187851716
115	1.84375	17.975397059954	15257	112.663851097912
116	1.84765625	18.422357232082	15709	116.200695063856
117	1.8515625	18.883381849167	16108	119.865694043801
118	1.85546875	19.358981589106	16505	123.664042651354
119	1.859375	19.849687868650	16931	127.601166378196
120	1.86328125	20.356053766723	17357	131.682732696659
121	1.8671875	20.878654992320	17731	135.914662735032
122	1.87109375	21.418090899408	18261	140.303143557937
123	1.875	21.974985550079	18744	144.854641077184
124	1.87890625	22.549988830770	19223	149.575913643017
125	1.8828125	23.143777622038	19740	154.474026338486
126	1.88671875	23.757057025786	20255	159.556366023061
127	1.890625	24.390561652940	20724	164.830657167220
128	1.89453125	25.045056974775	21351	170.304978522201
129	1.8984375	25.721340740147	21938	175.987780664290
130	1.90234375	26.420244464357	22519	181.887904477821
131	1.90625	27.142634991256	23149	188.014600614938
132	1.91015625	27.889416133468	23776	194.377549993856
133	1.9140625	28.661530395045	24364	200.986885395445
134	1.91796875	29.459960780838	25113	207.853214219665
135	1.921875	30.285732696269	25829	214.987642461042
136	1.92578125	31.139915944514	26540	222.401799987931
137	1.9296875	32.023626824285	27309	230.107867185561
138	1.93359375	32.938030334506	28078	238.118603047284
139	1.9375	33.884342491999	28815	246.447374799594
140	1.94140625	34.863832768081	29717	255.108189147762
141	1.9453125	35.877826649703	30595	264.115725229717
142	1.94921875	36.927708334053	31471	273.485369392684
143	1.953125	38.014923561949	32416	283.233251884200
144	1.95703125	39.140982598465	33364	293.376285575140
145	1.9609375	40.307463369346	34289	303.932206837072
146	1.96484375	41.516014761517	35386	314.919618698044
147	1.96875	42.768360096035	36469	326.358036405437
148	1.97265625	44.066300785181	37553	338.267935553732
149	1.9765625	45.411720182039	38721	350.670802914822
150	1.98046875	46.806587634114	39897	363.589190137032
151	1.984375	48.252962753023	41061	377.046770488458
152	1.98828125	49.752999912018	42405	391.068398823414
153	1.9921875	51.308952983609	43748	405.680174960415
154	1.99609375	52.923180332947	45100	420.909510692875
155	2.0	54.598150079695	46551	436.785200637558

Table 8 ($y' = x^3y$, seven nodes, 5 times) 4/4

S =step	$E=(\text{error of } y) \times 10^{12}$			
S	x	y	E	y'
156	2.00390625	56.336445644396	48018	453.337497158308
157	2.0078125	58.140771596384	49487	470.598189618386
158	2.01171875	60.013959820032	51150	488.600688221040
159	2.015625	61.958976017204	52826	507.380112714366
160	2.01963125	63.978926567354	54521	526.973386274945
161	2.0234375	66.077065764312	56334	547.419334874524
162	2.02734375	68.256803452294	58177	568.758792472850
163	2.03125	70.521713085349	60038	591.034712404293
164	2.03515625	72.875540234474	62111	614.292285337774
165	2.0390625	75.322211568309	64215	638.579064215606
166	2.04296875	77.865844337364	66355	663.945096623888
167	2.046875	80.510756389933	68636	690.443065045931
168	2.05078125	83.261476751839	70965	718.128435498828
169	2.0546875	86.122756804649	73334	747.059615090212
170	2.05859375	89.099582097474	75938	777.298119053608
171	2.0625	92.197184830051	78598	808.908747860727
172	2.06640625	95.421057049550	81315	841.959775069343
173	2.0703125	98.776964602589	84205	876.523146578044
174	2.07421875	102.270961888785	87167	912.674692022845
175	2.078125	105.909407465734	90198	950.494349105686
176	2.08203125	109.698980556615	93494	990.066401681695
177	2.0859375	113.646698515497	96881	1031.479732492361
178	2.08984375	117.759935311152	100353	1074.828091512468
179	2.09375	122.046441090627	104039	1120.210380912481
180	2.09765625	126.514362889839	107830	1167.730957725085
181	2.1015625	131.172266563651	111729	1217.499955386411
182	2.10546875	136.029160010553	115936	1269.633625384240
183	2.109375	141.094517772666	120276	1324.254700336163
184	2.11328125	146.378307099322	124744	1381.492779932855
185	2.1171875	151.891015564726	129477	1441.484741248196
186	2.12109375	157.643680338262	134363	1504.375175041111
187	2.125	163.647919213452	139409	1570.316849796272
188	2.12890625	169.915963506440	144820	1639.471205352108
189	2.1328125	176.460692943059	150422	1712.008878101555
190	2.13671875	183.295672663730	156210	1788.110259911835
191	2.140625	190.435192480578	162332	1867.966093027164
192	2.14453125	197.894308532224	168672	1951.778103397520
193	2.1484375	205.688887492625	175244	2039.759675060839
194	2.15234375	213.835653498583	182257	2132.136568369907
195	2.15625	222.352237972666	189541	2229.147685063556
196	2.16015625	231.257232532547	197092	2331.045883418369
197	2.1640625	240.570245187326	205069	2438.098846914765
198	2.16796875	250.311960037147	213355	2550.590010118213
199	2.171875	260.504200708589	221970	2668.819545756535
200	2.17578125	271.16997771836	231132	2793.105417237506
201	2.177734375	276.688024571403	235834	2857.623981854150
202	2.1796875	282.333660403766	240648	2923.784501169965
203	2.181640625	288.110156197960	245570	2991.632236388510
204	2.18359375	294.020852581822	250537	3061.213784808622
205	2.185546875	300.069182532578	255762	3132.577121718278
206	2.1875	306.258674109554	261039	3205.771643663851
207	2.189453125	312.592953274205	266435	3280.848213168230

Table 9 ($y' = x^3y$, nine nodes, 5 times) 1/5

$S=\text{step}$		$E=(\text{error of } y) \times 10^{12}$		
S	x	y	E	y'
1	-0.25	1.000977039492	0	-0.015640266242
2	-0.1875	1.000309038221	0	-0.006593833992
3	-0.125	1.000061037019	0	-0.001953244213
4	-0.0625	1.000003814705	0	-0.000244141556
5	0	1.000000000000	0	0.000000000000
6	0.0625	1.000003814705	0	0.000244141556
7	0.125	1.000061037019	0	0.001953244213
8	0.1875	1.000309038221	0	0.006593833992
9	0.25	1.000977039492	0	0.015640266242
10	0.3125	1.002387030245	23	0.030590424507
11	0.375	1.004956088681	46	0.052995731239
12	0.4375	1.009201161005	71	0.084510741754
13	0.5	1.015747708689	102	0.126968463586
14	0.5625	1.025344064453	146	0.182489214596
15	0.625	1.038883909402	215	0.253633766944
16	0.6875	1.057440095421	330	0.343616398195
17	0.71875	1.068995362366	157	0.396925859799
18	0.75	1.082314239140	111	0.456601319637
19	0.78125	1.097606892533	163	0.523379751460
20	0.8125	1.115108342938	158	0.598118415389
21	0.84375	1.135082582552	168	0.681818556896
22	0.875	1.157827506988	235	0.775653974408
23	0.90625	1.183680830389	175	0.881005608287
24	0.9375	1.213027195044	358	0.999503609197
25	0.96875	1.246306733651	198	1.133078732367
26	1.0	1.284025416832	144	1.284025416832
27	1.03125	1.326767589089	220	1.455079554721
28	1.0625	1.375211215329	203	1.649514819559
30	1.125	1.492498647915	311	2.125061551436
31	1.140625	1.526788892609	231	2.265727366012
32	1.15625	1.563356015901	254	2.416646492720
33	1.171875	1.602367228343	246	2.578730295018
34	1.1875	1.644004711491	241	2.752985428739
35	1.203125	1.688467192264	258	2.940525027031
36	1.21875	1.735971763395	282	3.142581343802
37	1.234375	1.786755554171	271	3.306520064060
38	1.25	1.841078539560	366	3.595856522577
39	1.265625	1.899225418624	293	3.850274107736
40	1.28125	1.961508701220	314	4.125645178125
41	1.296875	2.028271781646	320	4.424054859971
42	1.3125	2.099892470853	308	4.747828167131
43	1.328125	2.176786981066	342	5.099560946453
44	1.34375	2.259414427447	363	5.898859749614
45	1.359375	2.348281921575	362	5.898859749614
46	1.375	2.443950342179	463	6.353316221563
47	1.390625	2.547040882133	400	6.849612669512
48	1.40625	2.658242489227	422	7.392344568810
49	1.421875	2.778320331735	449	7.986685297798
50	1.4375	2.908125447971	431	8.850274107263

Table 9 ($y' = x^3y$, nine nodes, 5 times) 2/5

$S=\text{step}$		$E=(\text{error of } y) \times 10^{12}$		
S	x	y	E	y'
51	1.453125	3.048605760499	488	9.354276213446
52	1.46875	3.200818667403	513	10.141558731255
53	1.484375	3.365945460646	525	11.008748967441
54	1.5	3.545307861867	643	11.965414033780
55	1.515625	3.740387017603	594	13.022423708027
56	1.53125	3.952845357979	628	14.192147934599
57	1.546875	4.184551789328	686	15.488687197239
58	1.5625	4.437610782392	671	16.928141717498
59	1.578125	4.714396015477	766	18.528926579064
60	1.5859375	4.862510826524	792	19.396295240400
61	1.59375	5.017589353582	804	20.312141276305
62	1.6015625	5.180022403529	843	21.279592756846
63	1.609375	5.350226093351	877	22.302003892172
64	1.6171875	5.528643686249	902	23.382973020363
65	1.625	5.715747574109	879	24.526362149058
66	1.6328125	5.912041419465	962	25.736318197214
67	1.64062 5	6.118062469821	995	27.017296091562
68	1.6484375	6.334384061274	1026	28.374083900028
69	1.65625	6.561618326269	1058	29.811830186764
70	1.6640625	6.800419124744	1101	31.336073805147
71	1.671875	7.051485218236	1154	32.592776360334
72	1.6796875	7.315563708865	1189	34.668357598825
73	1.6875	7.593453767181	1182	36.489734008647
74	1.6953125	7.886010675268	1280	38.424360943427
75	1.703125	8.194150213178	1332	40.480278611838
76	1.7109375	8.518853421959	1375	42.666162321357
77	1.71875	8.861171776472	1432	44.991377389846
78	1.7265625	9.222232807137	1489	47.466039195001
79	1.734375	9.603246212643	1567	50.101078876661
80	1.7421875	10.005510509688	1623	52.908315261074
81	1.75	10.430420271180	1640	55.900533640854
82	1.7578125	10.879474008965	1763	59.091572109874
83	1.765625	11.354282762473	1843	62.496416225904
84	1.7734375	11.856579463166	1912	66.131302870156
85	1.78125	12.388229148723	2006	70.013834251084
86	1.7890625	12.951240111018	2088	74.163103117738
87	1.796875	13.547776070169	2205	78.599830363152
88	1.8046875	14.180169475869	2298	83.346516325870
89	1.8125	14.850936049327	2352	88.427607252697
90	1.8203125	15.562790690098	2520	93.869678544767
91	1.828125	16.318664885322	2645	99.701636592772
92	1.8359375	17.121725775896	2758	105.954941226893
93	1.84375	17.975397047609	2912	112.663851020537
94	1.8515625	18.883381836101	3043	119.865693960863
95	1.859375	19.849687854942	3223	127.601166290076
96	1.8671875	20.878654977970	3381	135.914662641615
97	1.875	21.974985534835	3500	144.854640976698
98	1.8828125	23.143777606044	3745	154.474026231731
99	1.890625	24.390561636165	3949	164.830657053854
100	1.8984375	25.721340722352	4143	175.987780542538

Table 9 ($y' = x^3y$, nine nodes, 5 times) 3/5

S =step	$E=(\text{error of } y) \times 10^{12}$			
S	x	y	E	y'
101	1.90625	27.142634972504	4397	188.014600485046
102	1.9140625	28.661530375299	4618	200.986885256976
103	1.921875	30.285732675349	4909	214.987642312538
104	1.9296875	32.023626802160	5184	230.107867026576
105	1.9375	33.884342468603	5420	246.447374629431
106	1.9453125	35.877826624912	5805	264.115725047218
107	1.953125	38.014923535683	6150	283.233251688506
108	1.9609375	40.307463341552	6495	303.932206627943
109	1.96875	42.768360066493	6927	326.358036180006
110	1.9765625	45.411720150638	7320	350.670802672344
111	1.984375	48.252962719774	7811	377.046770228647
112	1.9921875	51.308952948164	8304	405.680174680168
113	2.0	54.598150041905	8760	436.785200335237
114	2.0078125	58.140771556302	9405	470.598189293959
115	2.015625	61.958975974398	10020	507.380112363834
116	2.01953125	63.978926520111	10278	526.973385910530
117	2.0234375	66.077065718649	10672	547.419334496229
118	2.02734375	68.256803340509	10970	568.758792079494
119	2.03125	70.521713036624	11313	591.034711995933
120	2.03515625	72.875540184074	11711	614.292284912933
121	2.0390625	75.322211516258	12164	638.579063774317
122	2.04296875	77.865844283521	12511	663.945096164778
123	2.046875	80.510756334299	13002	690.443064568824
124	2.05078125	83.261476694261	13386	718.128435002218
125	2.0546875	86.122756745206	13891	747.059614574581
126	2.05859375	89.099582035870	14334	777.298118516181
127	2.0625	92.197184766246	14794	808.908747300926
128	2.06640625	95.421056983581	15347	841.959774487264
129	2.0703125	98.776964534318	15935	876.523145972228
130	2.07421875	102.270961818061	16442	912.674691391696
131	2.078125	105.909407392624	17089	950.494348449561
132	2.08203125	109.698980480766	17646	990.066400997138
133	2.0859375	113.646698436932	18316	1031.479731779294
134	2.08984375	117.759935229754	18955	1074.828090769523
135	2.09375	122.046441006179	19591	1120.210380137375
136	2.09765625	126.514362802367	20358	1167.730956917717
137	2.1015625	131.172266473068	21146	1217.499954545646
138	2.10546875	136.029159916493	21877	1269.633624506334
139	2.109375	141.094517675142	22753	1324.254699420846
140	2.11328125	146.378306998130	23552	1381.492778977817
141	2.1171875	151.891015459717	24468	1441.484740251630
142	2.12109375	157.643680229281	25382	1504.375174001117
143	2.125	163.647919100324	26281	1570.316848710725
144	2.12890625	169.915963388969	27349	1639.471204218665
145	2.1328125	176.460692821071	28434	1712.008876918041
146	2.13671875	183.295672537004	29484	1788.110258675584
147	2.140625	190.435192348943	30697	1867.966091735961
148	2.14453125	197.894308395396	31845	1951.778102048027
149	2.1484375	205.688887350505	33124	2039.759673651475
150	2.15234375	213.835653350760	34433	2132.136566895971

Table 9 ($y' = x^3y$, nine nodes, 5 times) 4/5

S =step	$E=(\text{error of } y) \times 10^{12}$			
S	x	y	E	y'
151	2.15625	222.352237818849	35724	2229.147683521492
152	2.16015625	231.257232372663	37228	2331.045881806957
153	2.1640625	240.570245021010	38753	2438.098845229211
154	2.16796875	250.311959864061	40269	2550.590008354531
155	2.171875	260.504200528599	41979	2668.819543912564
156	2.17578125	271.169997584342	43639	2793.105415306291
157	2.1796875	282.333660208576	45458	2923.784499148616
158	2.18359375	294.020852378633	47348	3061.213782693108
159	2.1875	306.258673897738	49223	3205.771641446660
160	2.19140625	319.075746577285	51368	3357.859202601094
161	2.1953125	332.502305921167	53552	3517.901800333685
162	2.19921875	346.570298686507	55760	3686.350529519113
163	2.203125	361.313486716312	58211	3863.683904824783
164	2.20703125	376.767557470215	60635	4050.409632705907
165	2.2109375	392.970241712393	63264	4247.066504405305
166	2.21484375	409.961438842501	66018	4454.226418603945
167	2.21875	427.783350397047	68777	4672.496543089523
168	2.22265625	446.480622282728	71881	4902.521625474080
169	2.2265625	466.100496342962	75059	5144.986463739997
170	2.23046875	486.692971907953	78309	5400.618548270652
171	2.234375	508.310978019343	81881	5670.190887836494
172	2.23828125	531.010557074125	85461	5954.525032992793
173	2.2421875	554.525032992783	89318	6254.494311404638
174	2.24609375	579.895358643709	93381	6571.027290651581
175	2.25	606.210061813603	97490	6905.111485345569
176	2.25390625	633.865760103267	102049	7257.797326648693
177	2.2578125	662.937276180894	106746	7630.202413660863
178	2.26171875	693.503937833695	111592	8023.516067736979
179	2.265625	725.649865182074	116874	8439.004212343982
180	2.26953125	759.464282374071	122231	8878.014602879118
181	2.2734375	795.041846879199	127978	9341.982432819860
182	2.27734375	832.483002882905	134050	9832.436344557038
183	2.28125	871.894358764931	140247	10351.004875599892
184	2.28515625	913.389090662453	147409	10899.423373216114
185	2.2890625	957.087374013704	154099	11479.116845130707
186	2.29296875	1003.116845130707	161421	12093.330761145207
187	2.296875	1051.613094997089	169356	12742.893918454892
188	2.30078125	1102.720197660269	177482	13430.473296933254
189	2.3046875	1156.591275773187	186173	14158.461071671713
190	2.30859375	1213.389106028944	195378	14929.409763971051
191	2.3125	1273.286767454497	204848	15746.043611297032
192	2.31640625	1336.468335755083	215159	16611.270785532232
193	2.3203125	1403.129627147221	225904	17528.196525795779
194	2.32421875	1473.478995397215	237124	18500.137257610078
195	2.328125	1547.738186062738	249733	19530.635775940122
196	2.33203125	1626.143252255404	261733	20623.477576057821
197	2.3359375	1708.945536587583	275081	21781.708423148860
198	2.33984375	1796.412724325239	289244	23012.653258947818
199	2.34375	1888.829973179993	303919	24317.936551980882
200	2.34765625	1986.501125601107	319804	25703.504206777545

Table 9 ($y' = x^3 y$, nine nodes, 5 times) 5/5

S	x	y	$E=(\text{error of } y) \times 10^{12}$	y'
201	2.3515625	2089.750009893464	3468	27174.647156979404
202	2.35546875	2198.921837007657	353885	28737.026777875894
203	2.359375	2314.384700386418	372660	30396.702265091398
204	2.36328125	2436.531186854647	392178	32160.160138572789
205	2.3671875	2565.780107192876	413002	34034.346044523740
206	2.37109375	2702.578355723206	453132	36026.699042369772
207	2.375	2847.472909012176	458206	38145.188579911159
208	2.37890625	3000.762974615232	483083	40398.354377034756
209	2.3828125	3163.202301679539	509246	42795.349457116334
210	2.38671875	3335.301666210034	536793	45345.986585936589
211	2.390625	3517.681544845735	566382	48060.788400054752
212	2.392578125	3612.932135196011	581638	49483.246725713306
213	2.39453125	1711.004992148743	597429	50951.041530995152
214	2.396484375	3811.990158382369	613687	52465.712607893348
215	2.3984375	3915.980737661625	630433	54028.855058178064
216	2.400390625	4023.073004891346	647675	55642.121382520793
217	2.40234375	4133.366520330664	665270	57307.223652215010
218	2.404296875	4246.964248133190	683710	59025.935766912298
219	2.40625	4363.972679381413	702628	60800.095801880933
220	2.408203125	4484.501959798718	721942	62631.608448559148
221	2.41015625	4608.666022321222	741946	64522.447552214912
222	2.412109375	4736.582724723228	762530	66474.658750744475
223	2.4140625	4868.373992500127	783755	68490.362218843716
224	2.416015625	5004.165967216643	561995	70571.755521907182
225	2.41796875	5144.089160540115	827981	72721.116584242525
226	2.419921875	5288.278614187463	851347	74940.806766383513
227	2.421875	5436.874066020056	875354	77233.274126435847
228	2.423828125	5590.020122538761	899909	79601.056660722688
229	2.42578125	5747.219880494120	925349	82046.785879100428
230	2.427734375	5910.567900655065	951523	84573.190370601674
231	2.4296875	6078.284825693147	978533	87183.099575350096
232	2.431640625	6251.183156347517	1006376	89879.447698871510
233	2.43359375	6429.434622299452	1034901	92665.277785254802
234	2.435546875	6613.217206399724	1064644	95543.745955886819
235	2.4375	6802.714869800313	1095240	98518.125820723825
236	2.439453125	6998.118285879149	1126586	101591.813069493389
237	2.44140625	7199.624833312938	1159069	104768.330250414697
238	2.443359375	7407.438898670728	1192496	108051.331744410476
239	2.4453125	7621.772138928422	1227011	111444.608943188855
240	2.447265625	7842.843754286429	1262617	114952.095639845900
241	2.44921875	8070.880771751216	1299147	118577.873641100556
242	2.451171875	8306.118339888319	1337178	122326.178610657629
243	2.453125	8548.800035225272	1376340	126201.406153556946
244	2.455078125	8799.178180783736	1416525	130208.118151940666
245	2.45703125	9057.514177239463	1458171	134351.049362995487
246	2.458984375	9324.078847232552	1501046	138635.114290356846
247	2.4609375	9599.152793377393	1545339	143065.414340832050
248	2.462890625	9883.026770537452	1591064	147647.245278712817
249	2.46484375	10176.002072960939	1638038	152386.104990593099
250	2.466796875	10478.390936897274	1686885	157287.701574160260